



MATHEMATICAL MODELING TO PREDICT GLOBAL WARMING THROUGH HUMAN ACTIVITY

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Abstract

The objective of this research work is to predict global warming through the development of a mathematical model, taking as a reference human activity in the period from 1880 to 2016. It is assumed that global human activity is in function of the Internal Product Gross and population growth. To evaluate this relationship, the operations research methodology was used, this tool allowed the formulation, construction, solution, validation and implementation of the model. With the obtained results it was possible to appreciate the different stages that have experienced the increase of the global average temperature; as a consequence, an irrational exploitation of the natural resources. It was demonstrated that the global average temperature has increased to 0.90°C in 136 years; as a consequence, this behavior is the result of the increase of human activity in 11.21 units.

Keywords: Global warming, Operations research and Mathematical modeling.

INTRODUCTION

To start with, according to Intergovernmental Panel on Climate Change (IPCC), global warming is the result of the increase of greenhouse gases mainly carbon dioxide. Such event can be considered as a social fact, in which human activity has intervened; for this reason, it is supported into two elements: wealth accumulation through GGDP and population growth because both have caused an over-use of habitat (Guerrero, 2017). In the last five decades, the increase of the level of greenhouse gases through the human activity have led to global warming of 0.5°C. According to IPCC, from 1880 to 2016, the average temperature of the planet has increased to 0.86°C. If this trend continues, the projections will reflect a temperature increase to the year 2100 between 1.8 and 4°C. This creates an amount of uncertainties in the future with potential environmental, economic, social and health consequences (Ministry of Health, 2013). In this context, it is utmost importance the development of the studies about the global warming, thus there are strong variations in the ecosystems, the estimation of the average temperature of the planet will become a priority, and more over when human activity grows rapidly. Developing a mathematical model which foresees global warming on the basis of human activity, considering the relevant and innovative time. Currently, some models of cross section, this is the case of IPAT equation which was built by the Biologist Barry Commoner, and whose main objective is to study the human impact over the environment, taking as a reference population growth and technology used (Gitay *et al.*, 2002). The current model has as objective to predict the variation of the global average temperature since then the dynamic of human activity in a period of 136 years from 1880 to 2016. Taking as a basis the records of global temperature of the Environmental Protection Agency (EPA) from United States. To predict the global warming involves to say that the variation of the average temperature of the planet is in function of the human activity (Guerrero, 2017):

AT = f(HA) (1)

Where:

AT is the variation of the global average temperature in the time.

HA is the human activity in the time.

HA = f(GGDP, WTP) (2)

HA = f(GGDP, WTP) → At = f(HA) (3)

Human activity in time is in function of the GGDP and WTP, where:

GGDP is Global Gross Domestic Product

WTP is the World's Total Population

GGDP is the sum of the goods and services produced during a specific time (regularly in a year) generally speaking (Gregory, 2012). Meanwhile, the global population is the total number of inhabitants who are registered in the planet (Pressat, 1987).

Examples:

Based on the above, the hypothesis of the variation of the global average temperature are the following:

“Greater Human activity, greater variation of the global average temperature”

“Greater GGDP, greater Human Activity”

“Greater World Population, greater Human Activity”

In order to predict global warming through the construction of a mathematical model can monitor the global average temperature and the possible effects such as droughts, hurricanes, floods, hunger, poverty and malnutrition, among others. Taking as a reference human activity over natural resources, because of a large part of the world economy is sustained through the irrational exploitation of natural resources.

METHODS

The construction of the model was based on the Research methodology of operations, which must have fulfilled each stage (Taha, 2004):

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1. Problem statement: the relation between the global average temperature and human activity are outlined.
2. Construction of the model: the mathematical proposal was taken into account from the existing relations between the global average temperature and human activity.
3. Model solution: since then the mathematical proposal, different operations were used to find the equations required to predict the global average temperature. Added to this, the mathematical expectation and the confidence interval.
4. Model validation: in this stage the verification of the equations was done through the application of verified data.
5. Implementation: the interpretation of the results was obtained through the equations, the mathematical expectation and the confidence interval.

Taking as a reference the human activity during the period from 1880 to 2016, the population of subject of the study considers the global average temperature.

Modeling

Problem statement: It is known that the variation of the global average temperature functions according to human activity in the time:

$$HA = f(GGDP, WTP) \rightarrow At = f(HA) \tag{4}$$

The rate of human activity will be the same as the natural logarithm of the GGDP and the natural logarithm of Wtp:

$$HA = \ln(PIB) + \ln(Pt) \rightarrow \text{doing symmetrical information} \tag{5}$$

In accordance with the report of Global Climate Change broadcasted by EULA-Chile Center in 2016, global warming has increased exponentially, thus the global average temperature has raised to 0.86°C from 1880 to 2016 (Global Climate Change, 2017):

Where:

ΔHA is the exponential increase of the global average temperature in function to the human activity in the time.
 HA is the global average temperature when there is no intervention of the human activity in the time.

The interpretation of the variation of the global average temperature is the following: for each percentage point that the GGDP and WTP have increased in the time, HA and AT will increase too. In other words, as long as the way human activity incentivizes, the global average temperature will raise. This behavior is sustained by the study of Gitay, in which shows the difficulty to qualify global warming taking into consideration natural causes because human activity is seen as the main cause, this confirms the Fifth Assessment Report (GTI IE5), from 2014, provides as follows:

It is highly likely that human influence has been the main cause of global warming since by mid of the twentieth century, this is considered as consequence of the technological advances which have required the irrational exploitation of natural resources (IPCC-GT II, 2014). Where the information of GGDP and WTP were obtained from World Bank:

GGDP in billions of American dollars from 1880 to 2016.
 WTP in billions of people from 1880 to 2016.

Construction of the model: Based on the charts 1 and 2, the equation of the global average temperature is the following way (Montgomery and Runger, 2007):

$$AT = (\beta + \beta_0 HA)^2 \tag{6}$$

Taking as a reference the algebraic expression (6), the mathematical model of the global average temperature will be the following way (Montgomery and Runger, 2007):

$$AT = \beta^2 + 2\beta\beta_0 HA + \beta_0^2(HA)^2 \tag{7}$$

Where:

β^2 is the expected value of the global average temperature when there is no intervention of human activity in the time.

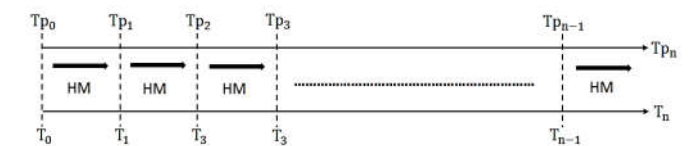
$\frac{dAt}{dHM}$ is the marginal change that experiments the global average temperature before a change of human activity in the time.

$$\frac{dAt}{dHA} = 2\beta\beta_0 + 2\beta_0^2 HA = 2\beta_0(\beta + \beta_0 HA) \tag{8}$$

The functionality of the equation (7) consists in the determination of the rate of HA :

$$HA = \ln(GGDP) + \ln(WTP) \tag{9}$$

Through numerical sequences or series (Bruzual and Domínguez, 2005), the effects towards the global average temperature is the following way (Guerrero, 2017):



Source: Own authorship

Figure 1. Dynamic of the rate HA in the time

In the time T_0 has a temperature AT_0 , to the time T_1 coexists AT_1 . If HA is time constant, At , will have the following behavior (Guerrero, 2017):

$$\begin{aligned} At_1 &= At_0 + HA * At_0 = At_0(1 + HA) && \text{First moment} \\ At_2 &= At_1 + HA * At_1 = At_1(1 + HA) = At_0(1 + HA)^2 && \text{Second moment} \end{aligned}$$

Thus:

$$At_n = At_0(1 + HA)^n: n = 1,2,3, \dots \dots k \text{ moments} \tag{10}$$

As it can be seen, the global average temperature converges to the infinite. In summary, the global average temperature has an exponential behavior, which is in function of the dynamic of human activity in time. In other words, in the moment “n” human activity has a specific behavior that will have an impact on the increase of the global average temperature, having as a result the presence of climatological phenomena for example, droughts, floods, frosts, hurricanes, high temperatures among others.

Model solution

It is known that:

$$At = \beta^2 + 2\beta\beta_0HA + \beta_0^2(HA)^2, y \text{ que } HA = \ln(GGDP) + \ln(WTP) \quad (11)$$

If the time plays an essential role in At since the rate of HA is time constant, that means:

$$HA = \beta_1 + \beta_2 T \quad (12)$$

Where:

β_1 & β_2 are the parameters to estima

Through the Matrix system, the values of β_1 & β_2 are the following ones (Lipschutz and Lipson, 2009):

Through determinants (Grossman, 2007):

$$\beta_1 = 8.40 \& \beta_2 = 0.09$$

Therefore:

$$HA = 8.40 + 0.09T \quad (13)$$

If it known that $At = f(HA)$ and as y HA is implicit to T, then:

$$At = \beta^2 + 2\beta\beta_0HA + \beta_0^2(HA)^2 \quad (14)$$

Where:

$$\beta^2 = A_0 \quad 2\beta\beta_0 = A_1 \quad \beta_0^2 = A_2$$

Substituting:

$$At = A_0 + A_1HA + A_2(HA)^2 \quad (15)$$

Through the determinants:

$$A_0 = 14.87; \quad A_1 = -0.21 \quad \& A_2 = 0.01$$

Substituting (12) in (15):

$$At = 14.87 - 0.21(8.40 + 0.09T) + 0.01(8.40 + 0.09T)^2$$

Substituting terms:

$$At = 13.82 - 0.004T + 0.00008T^2 \quad (17)$$

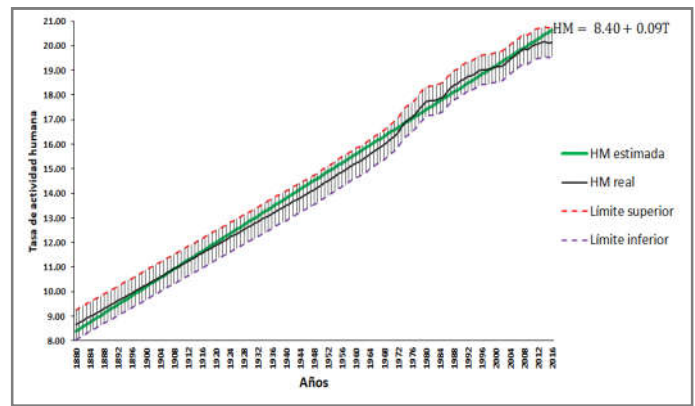
Through the algebraic expressions (13) and (17), we can obtain a specific conjecture about the dynamic of human activity in the time. It is decisive to the behavior of the global average temperature.

$$At = 14.87 - 0.21HA + 0.01(HA)^2 \quad (16)$$

Model validation

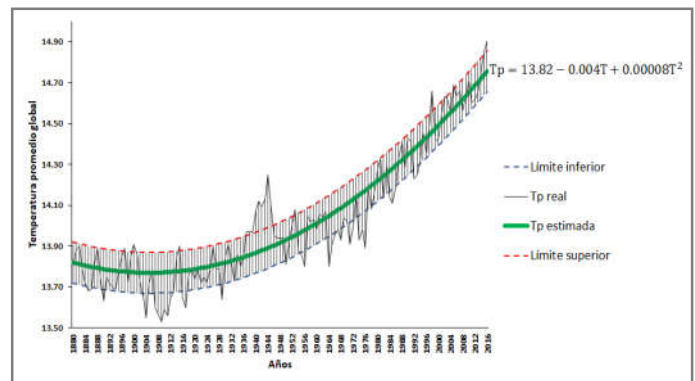
Through the Figure 2 and 3, we can observe the balance equations (12) and (16). In the figure 4, HA is calculated (Green straight) is lightly found over the real data HA(black straight). Besides this, all the estimated data is located within a confidence interval; as a result, it has a proper adjustment.

In the figure 3, the parable describes the equation (16), it represents the expected value of the global average temperature in the time (Green parable), and thus its degree of adjustment is good. Because of the majority of the real data (black straight) is found within its confidence interval.



Source: Own authorship

Figure 2. Estimate of Human Activity in the time



Source. Own authorship

Figure 3. Estimate of the global average temperature in the time

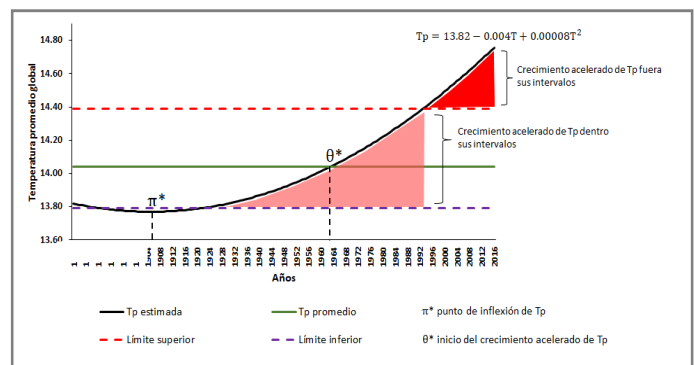
Interpretation of the results

Based on the equations (12) and (16), the interpretation of its parameters is the following:

$$HA = 8.40 + 0.09T$$

- The expected value of the human activity is of 8.40 units when the time remains constant.
- For each elapsing year, the human activity will increase to 0.09 units: $\frac{dHA}{dT} = 0.09$
- The expected value of the global average temperature is 13.82°C when the time remains constant.

From the figure 6 and the equation (16), it can be observed that the global average temperature decreases to 0.03°C from 1880 to 1905. This represents an increase of 1.13 units over the human activity during the last 25 years (Thomas, 1974).



Source: own authorship

Figure 4. Dynamic of the global average temperature in the time

From 1905 (π^*) to currently the trend has been growing, this behavior is divided into stages. Firstly, the growth is found within the intervals of the global average temperature. Secondly, the growth exceeds the upper limit.

<p>Temperature behavior (1880-1905)</p> $\frac{dAt}{dT} = -0.004 + 0.00016T$ <p>If $\frac{dAt}{dT} = 0$, so:</p> $T = \frac{0.004}{0.00016} = 25 \text{ years}$ <p>Looking for the A_{t_1}:</p> $\bar{A}_{t_1} = \frac{1}{25} \int_{1880}^{1905} (13.82 - 0.004T + 0.00008T^2) dT$ <p>Solving definite:</p> $\bar{A}_{t_1} = \frac{1}{25} [344.67] = 13.79^\circ\text{C}$ <p>Average variation of temperature:</p> $\Delta A_{t_1} = 13.79 - 13.82 = -0.03^\circ\text{C}$	<p>Human activity behavior (1880-1905).</p> $\bar{H}A_1 = \frac{1}{25} \int_{1880}^{1905} (8.40 + 0.09T) dT$ <p>Solving definite integral:</p> $\bar{H}A_1 = \frac{1}{25} [238.13] = 9.52 \text{ units}$ <p>Average variation of human activity:</p> $\Delta H A_1 = 9.52 - 8.40 = 1.13 \text{ units}$
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First stage

Based on the results obtained in the current model, the global average temperature is approximately 14.04°C, which its dynamic ranges between 13.82°C to 14.39°C. On the basis of this information, it is observed on the chart 6 that from 1905 to 1993 the global average temperature increased to 0.24°C.

<p>Temperature behavior (1905-1993)</p> $14.39 = 13.82 - 0.004T + 0.00008T^2$ <p>Reducing terms and applying the general:</p> $T = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ <ul style="list-style-type: none"> • 0.00008 • -0.004 • -0.57 <p>Therefore:</p> $T = 113.03 \sim 113 \text{ years}$ <p>This result stays from the period 1880 to 1993. Looking for A_{t_2} from 1905 to 1993:</p> $\bar{A}_{t_2} = \frac{1}{88} \int_{1905}^{1993} (13.82 - 0.004T + 0.00008T^2) dT$ <p>Solving the definite integrals:</p> $\bar{A}_{t_2} = \frac{1}{88} [1234.69] = 14.03^\circ\text{C}$ <p>Average variation of temperature:</p> $\Delta A_{t_2} = 14.03 - 13.79 = 0.24^\circ\text{C}$	<p>Human activity behavior (1905-1993).</p> $\bar{H}A_2 = \frac{1}{88} \int_{1905}^{1993} (8.40 + 0.09T) dT$ <p>Solving the definite integral:</p> $\bar{H}A_2 = \frac{1}{88} [1285.67] = 14.61 \text{ units}$ <p>Average variation of human activity: $\Delta H A_2 = 14.61 - 9.52 = 5.09 \text{ units}$</p>
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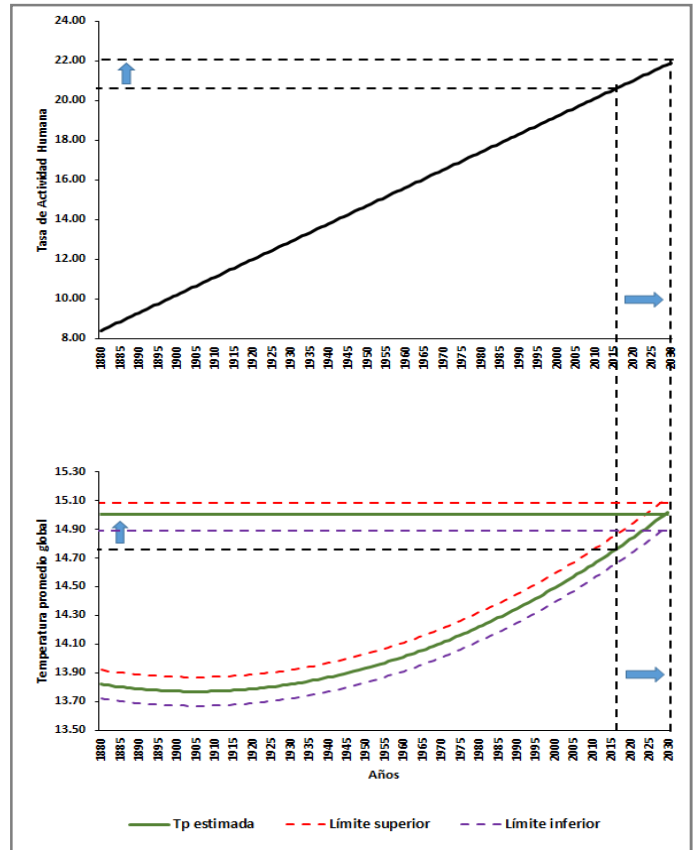
This increase of 0.24°C has enhanced of 5.09 units over the human activity in a period of 88 years.

Second stage

From 1993 to 2016, the increase of global average temperature has getting out of the confidence intervals because this has registered to 14.72°C. In these 23 years the global average temperature has increased to 0.69°C. It shows an increase in human activity of 4.99 units. This setting might be considered as a space where phenomena of climate change occur. Taking as a reference the context of the model, the increase of global average temperature has been of 0.90°C from 1880 to 2016, this causes an accelerated growth in human activity. In the figure 4, an increase in the global average temperature is observed and emphasized with a greater speed from 1963 (6*). This behavior takes into account the accelerated growth lived by the population and a large scale economy (human activity).

<p>Temperature behavior (1993-2016)</p> <p>Looking for the A_{t_3} from 1993 to 2016:</p> $\bar{A}_{t_3} = \frac{1}{23} \int_{1993}^{2016} (13.82 - 0.004T + 0.00008T^2) dT$ <p>Solving the Definite integral:</p> $\bar{A}_{t_3} = \frac{1}{23} [338.58] = 14.72^\circ\text{C}$ <p>Average variation of the temperature:</p> $\Delta A_{t_3} = 14.72 - 14.03 = 0.69^\circ\text{C}$	<p>Human activity behavior (1993-2016).</p> $\bar{H}A_3 = \frac{1}{23} \int_{1993}^{2016} (8.40 + 0.09T) dT$ <p>Solving the Definite integral:</p> $\bar{H}A_3 = \frac{1}{23} [450.91] = 19.60 \text{ units}$ <p>Average variation of human activity:</p> $\Delta H A_3 = 19.60 - 14.61 = 4.99 \text{ units}$
<p>General increase of the global average temperature</p> $\Delta A_{t_g} = \sum_{i=1}^3 \Delta A_{t_i} = \Delta A_{t_1} + \Delta A_{t_2} + \Delta A_{t_3}$ <p>Therefore:</p> $\Delta A_{t_g} = -0.03 + 0.24 + 0.69$ $\Delta A_{t_g} = 0.90^\circ\text{C}$	<p>General increase of the human activity</p> $\Delta H A_g = \sum_{i=1}^3 \Delta H A_i = \Delta H A_1 + \Delta H A_2 + \Delta H A_3$ <p>Therefore:</p> $\Delta H A_g = 1.13 + 5.09 + 4.99$ $\Delta H A_g = 11.21 \text{ units}$

To continue this trend to the year 2030, human activity will be approximately of 22 units, this equals to a global average temperature of 15°C and ranges between 14.90 and 15.10°C. An increase of 1.18°C would represent during a period of 150 years (figure 5). These results follow a series of events such as:



Source: Own authorship

Figure 5. Prediction of the global average temperature in 2030

Conclusion

It is necessary to mention that with the development of the current mathematical model it was possible to know the dynamic of global warming, taking as a reference human activity. In doing so, it was demonstrated that the global average temperature has increased to 0.90°C in 136 years; as a consequence, this behavior is the result of the increase of human activity in 11.21 units. From 1963, the global average temperature has experienced an accelerated growth. In 1993, however, the effects have been shown through the presence of climatic phenomena for instance hurricanes, floods, and droughts, among others. From the date of the global average temperature has been out of its intervals. This is the result of the accelerated growth of human activity, which has been sustained in the irrational exploitation of natural resources through the greenhouse gases emission. By using this model, the global average temperature will have an approximated behavior of 15°C in 2030 due to human activity will be of 22 units, this would show an increase of 1.18°C in a period of 150 years. With this model, it was possible to observe the degree of influence that human activity has had over global warming in the time. Based on this context, it is crucial that more mathematical and probabilistic models continue being developed, which their main purpose is to diagnose climate change. Furthermore, the coming research work will predict the effects that economic growth can produce over global

warming, that means, what is the maximum economic growth that can be shown without affecting the global average temperature?

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