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Research Article

CALCULATION OF THE NON–POSITIVE SECTIONAL CURVATURE USING MATLAB

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Abstract

The study of sectional curvature is considered an important study as it is used in many sciences such as engineering, physics and mathematics. Sectional curvature has several types, including positive, non-positive (negative) and non-negative sectional curvature. This study deals with non-positive sectional curvature and is considered an addition to what was presented in this field .The aims of this paper is to calculate the non-positive sectional curvature by using Matlab. It also aims to find some applications of non-positive sectional curvature .We followed the applied mathematical method using Matlab. Also we showed the calculation of Non –Positive Sectional Curvature using Matlab and their some applications by following descriptive approach in order to achieve the stated objectives of the paper, and we found the following some results: Matlab gives precise results of high speed compared with that of Manual, Also stated the ability and capability of Graphs or Diagram drawing to any curvature via Matlab. Explained the possibility of the calculation of Non –Positive Sectional Curvature by Matlab with a very high rate and accuracy also shed lights on as Cartan –Hadamard theory which is considered one of the most important applications regarding Non –Positive sectional curvature.

Keywords: Single Project, Implementation Unit, and Donor Funded Project.

1. INTRODUCTION

In 1928 Elie Cartan proved the Cartan- Hadamard theorem : if μ is a complete manifold with non-positive sectional curvature then its universal cover is diffeomorphic to a Euclidean space in particular it is a spherical the homotopy groups $\pi_i\ (\mu)$ for i≥ 2 are trivial therefore the topological structure of complete non-positively Curved manifold is determined by its fundamental group preissman`s theorem restricts the fundamental group of negatively curved compact manifold the Cartan-Hadamard conjecture states that the classical isoperimetric inequality should hold in all simply connected space of non-positive curvature which are called Cartan-Hadamard manifold.

Theorem(1.1):

(Cartan- Hadamard). Let (μ, g) be a complete Riemannian manifold with non-positive sectional curvature, then for any p ∈ μ, the exponential map exp $_p$: T $_p\mu \rightarrow \mu$ is a covering map. In particular, if μ is also simply connected, then exp $_p$ is a differmorphism (and thus μ is non-compact). Before we prove the Cartan -Hadamard theorem, we need the following lemma saying that any non-positive curvature manifold has no conjugate point. (Recall we showed this for non-positive constant curvature case by explicitly computing the Jacobi fields.)

Example(1.2):

Let μ be a Riemannian manifold of non-positive sectional curvature, I, e. K($\prod$) ≤ 0 for any 2-plane $\prod \subset$ TM .

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(a) Let c : $[a, b] \rightarrow$ M be a geodesic and let J be a Jacobi field along c. Let f (t) =$\|J(t)\|^2$. Show that $f^{//}$(t) ≥ 0, I .e ., f is a convex function .
(b) Derive from (a) that M does not a dmit conjugate points .

Solution

$$f^{/}\ (t) = \frac{d}{dt}\Big|_{t=0} \langle J(t), J(t)\rangle = 2\ \langle \frac{D}{dt}J(t), J(t)\rangle$$

and

$$f^{//}(t) = 2(\ \langle \frac{D^2}{dt^2}\ J(t), J(t) + \ \left\|\frac{D}{dt}J(t)\right\|^2\rangle)$$

Using Jacobi equation, we conclude

$$f^{//}(t) = 2(-\ \langle R(J(t), c^{\backslash}(t)c^{\backslash}(t), J(t)\rangle + \left\|\frac{D}{dt}J(t)\right\|^2)$$

We have $\langle R\left(J(t), c^{\backslash}(t)\right) c^{\backslash}(t), J(t)\rangle = 0$ if J(t), $c^{\backslash}(t)$ are linear dependent and, otherwise, for $\prod$ = span (J(t), $c^{\backslash}(t) \subset T_{c(t)}M)$.

$$\langle R\left(J(t), c^{\backslash}(t)\right) c^{\backslash}(t), J(t)\rangle = \text{K}\ (\textstyle\prod)(\|J(t)\|^2\|c^{\backslash}(t)\|^2\text{-}(\langle J(t), c^{\backslash}(t)\rangle^2) \leq 0,$$

Since sectional curvature is non-positive. This shows that $f^{//}$(t), as a sum of two non-negative term, is greater than or equal to zero.

If there were a conjugate point q = c(t_2) to point p = c(t_1) along the geodesic c, then we would have a non –vanishing Jacobi field J along c with *J*(t_1) *= 0* and *J*(t_2) *= 0*. This would imply that the convex, non – negative function *f (t)* = $\|J(t)\|^2$ would have zeros at $t = t_1$ and *t*= t_2. This would force *f* to vanish identically on the interval $|t_{1,}t_2|$, which would imply that *J* vanishes as well, which leads to a contradiction [4] .

Corollary (1.3):

Let *(μ, g)* be a complete simply connected flat manifold then *(μ, g)* is isometric to (IR^n, g_0).

Proof:

Choose any *p*. Identify $T_p M$ with IR^n and let $\bar{g}$ = exp$*_p$ g on IR^n. We have already proved that the map

$Exp_p : (IR^n, \bar{g}) \rightarrow (\mu, g)$

is both a diffeomorphism and a local isometry. So it is a global isometry. Since *g* is flat, $\bar{g}$ is then a flat metric on IR^n. But two flat metrics on IR^n differ only by a linear isomorphism.

2. Approach to Hyperbolic in an Integral Sense

Recall that the goal of the cross curvature flow is to deform a metric with negative sectional curvature on a 3-manifold to a hyperbolic metric. We show that an integral measure of the difference of the metric from hyperbolic is monotone decreasing. Let

$$J = \int_m (\frac{p}{3} - (det\ P)\ 1/3\ d\mu \quad (1)$$

where $P = g_{Ij} P^{Ij}$. By the arithmetic-geometric mean inequality (applied in a basis in which P_{Ij} is diagonal and $g_{Ij} = \delta_{Ij}$), the integrand is nonnegative, and identically zero if and only if $P_{Ij} = 1/3P\, g_{Ij}$, *i.e.*, g_{Ij} has constant curvature.[5]

Theorem(2.1) : Under the cross curvature flow $\frac{dj}{dt} \leq 0$.

Proof:

We compute

$$\frac{d}{dt}\int_m P\ d\mu\ = \int_m [(\partial t g_{ij})\ p_{ij}\ + g_{ij}\partial t p_{ij}\ +\ P\ H\ ]d\mu$$

$$= \int_m [2h_{ij}p_{ij}\ + g_{ij}\ (-\ det\ P\ g_{ij}\ -\ H\ p_{ij})\ +\ P\ H\ ]d\mu$$

$$= 3\int_m det\ P\ d\mu.$$

By the definition of h_{Ij} we can replace *det P* by *(det h) 1/3 (det P) 1/3*. when $\eta = 1/3$ we find that

$$\frac{dj}{dt} = -\frac{1}{6}\int_m (\ \lfloor E_{ijk} - E_{jik}\rfloor^2\ +\ \frac{1}{3}\ \lfloor T_i\rfloor^2)\ (det\ P)\ 1/3\ d\mu$$

$$-\int_m (\frac{H}{3}\ -\ (det\ h)1/3)\ (det\ P)\ 1/3\ d\mu.$$

This is non positive (and if and only if g_{Ij} has constant negative sectional curvature).

i. A maximum Principle Estimate

We can also obtain information about the long-time behavior of geometric flows like the *XCF* by using the maximum principle for parabolic equations. That typically involves finding a function *f(x, t)*, constructed tensorially from the metric and its derivatives, which satisfies an inequality of the form $\partial t f \geq \Delta f$. Since the higher order terms in the evolution of P_{Ij} are of divergence form with both first and second order terms, in comparison to the Ricci flow, it is much more difficult to obtain good maximum principle estimates. But, as we show next, there is at least one function which, under the *XCF*, satisfies an equation for which the maximum principle can be applied. At the end of this section we use the maximum principle to show that the *XCF* preserves the set of metrics of negative sectional curvature unless singularities arise in finite time [3].

Lemma(2.2): Let (μ^n, g) be Riemannian manifold

(a) If the Ricci curvature is positive, then the identity map $I : (\mu, g_{Ij}) \rightarrow (\mu, g_{ij})$ is harmonic, and if the Ricci curvature is negative then $I : (h, g_{Ij}) \rightarrow (\mu, g_{Ij})$ is harmonic.

(b) *If n = 3* and the sectional curvature is negative (or positive), then

$I : (\mu, g_{Ij}) \rightarrow (\mu\ g_{Ij})$ is harmonic. *[18] pp23.*

3. Parallelism and Geodesics

Having introduced two fundamental notions of Riemannian geometry, the Riemannian metric tensor and its associated Riemannian connection, we are now in a position to examine some geometric notions beyond those of length and angle introduced in Sect. In particular, we will define geodesics, the analogue of "straight lines" in the context of "curved space." This concept relies heavily on the notion of the covariant derivative introduced .In order to generalize the notion of a line to the setting of a Riemannian space, we first need to have a sense of what qualities of a line we hope to generalize. In fact, as with many fundamental notions, the concept of a line unites a number of seemingly distinct properties. In Euclid's axiomatic geometry, a line is completely described by two distinct points. In this setting, two lines are parallel if they have no point of intersection. In the analytic description of Euclidean geometry, a (non vertical) line is described by one point and a number, the slope; two distinct lines are parallel if they have the same slope.

In the vector geometry of $R^{3,}$ a line can be characterized by one point (represented by a position vector) and a unit "direction" vector. Two lines can then be said to be parallel if their direction vectors are the same, up to sign. Key to this notion is the ability to compare the direction vectors at different points in IR^3. Another property of a Euclidean line might be offered by physics. A particle's motion is linear if its acceleration at each point is zero. This is essentially Newton's first law. For the notions introduced in this section, we will consider vector fields V defined along a curve. Let $c: I \rightarrow R^n$ be a smooth parameterized curve. A vector field alongcis a map $V: I \rightarrow T\, IR^n$, given by $T \rightarrow V(t) \in Tc(t)\, IR^n$, which is smooth in any of the obvious senses from that the component functions $Vi(t)$ relative to a coordinate basis of $T\,c(t)\,IR^n$ are smooth functions of t. Such might be the case, for example, for a vector field V defined on IR^n and then restricted to a parameterized curve $c : I \rightarrow IR^n$, i.e., $V(t) = V(c(t))$ for all $t \in I$. However, vector fields along a curve need not arise in this way. For a smooth parameterized curve $c : I \rightarrow IR^n$, the velocity vector fieldc (t)=(c ∗) $\frac{dc}{dt}$ (also denoted by $\frac{dc}{dt}$), where $\frac{dc}{dt}$ is the standard basis vector field on $T\, IR^1$, is a case in point.

In coordinates,
$c(t) = \sum_{i=1}^{n} \frac{dc}{dt} \frac{\partial}{\partial_{xi}}$, when c (t) = $(c_1(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots c_n(t))$.
This vector field is not defined for points not on c (I) It is not hard to see that the definition of the covariant derivative ∇ x y extends directly to vector fields x, y along a curve c. With this in mind, we define the derivative of a vector field V along the curve c to be

$$\frac{DV}{dt} = \nabla_{i(t)} V$$

We obtain

$$\frac{DV}{dt} = \sum_{i=1}^{n} \left(\frac{dv^k}{dt} + \sum_{i,j} \Gamma_{i,j}^{k}, \frac{dc_i}{dt}\right)) \frac{\partial}{\partial k_x},$$

Where V (t) = $\sum v^i \frac{\partial}{\partial_{xi}}$. Note that when V is a vector field on *IR* ^{n} restricted to a curve *c : I → IR* ^{n}, we have

$$\frac{dv^k}{dt} = \sum \frac{\partial v^k}{\partial x_i} \frac{dx_i}{dt},$$

Definition (3.1): Let V be a vector field along a parameterized curve

c : I → IR^n. Then *V* is parallel along *c* if $\frac{Dv}{dt} = 0$ for all $t \in I$.

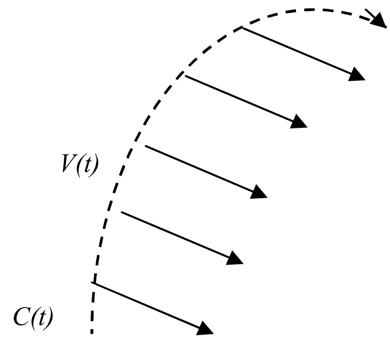


Figure 3.1. A parallel vector field *V* along a curve c with the standard Euclidean metric

The terminology is suggestive here: *V* is parallel along c if it is "constant along c." We will see shortly that this version of "constant" is closely related to the geometry and, specifically, to the metric tensor. The condition for v to be parallel along c can be expressed, using coordinates, by saying that the components v $_I$ of v are solutions to the first-order system of differential equations

$$\begin{cases} \frac{dv^{-1}}{dt} + \sum_{i,j} \Gamma_{ij}^{1} \frac{dc_i}{dt} v^j = 0 \\ \frac{dv^n}{dt} + \sum_{ij} \Gamma_{ij}^{n} \frac{dc_i}{dt} v^j = 0 \end{cases}$$

We illustrate the impact of the metric tensor on this notion of parallelism with two examples [1] pp224to226.

4. Geodesics and Curvature

Let $Y = (a, b) \to x$ be a smooth path. The energy of Y is

$$E\ (Y) = \frac{1}{2} \int_a^b \left| Y * \left(\frac{\partial}{\partial t}\right) \right|^2 dt$$

And the length is

$$L\ (Y) = \int_a^b \left| Y * \left(\frac{\partial}{\partial t}\right) \right| dt$$

A geodesic is a path which locally minimizes the length in the following sense. A variation of Y is a function $F = (-\epsilon, \epsilon) \times (a, b) \to X$ so that $F(0, t) = Y(t)$. The infinitesimal variation of Y corresponding to F is the vector field along $Y, S = F * \left(\frac{\partial}{\partial s}\right)$. We denote by T is the tangent vector field along $Y, T = Y * \left(\frac{\partial}{\partial t}\right)$. We have the following two important formulae. The first variational formula for the energy:

Lemma (4.1):

$$\frac{\partial}{\partial s} E(Y_s) = -\int_a^b < S, \nabla_T T > dt + < S, T > |_a^b$$

And the first variational formula for the length:

$$\frac{\partial}{\partial s} L(Y_s) = -\int_a^b < S, \nabla T/|T|^T > + < S, T/|T| > |_a^b$$

Proof.

$$\frac{\partial}{\partial s} E(Y_s) = \int_a^b < S, \nabla_s T > dt$$

$$= \int_a^b < S, \nabla_T S > dt$$

$$= \int_a^b \left(\frac{\partial}{\partial t} < T, S > - <, \nabla_T T >\right) dt$$

$$= < S, T > |_a^b - \int_a^b <, \nabla_T T, S > dt$$

The proof for the length is similar and left to the reader. As a consequence we have that Y is a geodesic if and only if

$$\nabla_T (T/|T|) = 0$$

In other words the unit tangent vector to y is parallel along Y if we parameterize Y proportional to are length then $\nabla_T T = 0$. We also have the second variational formula [2] pp 20 to 23.

Definition (4.2): By definition, of n –dimensional manifold of constant curvature κ is a length space X that is locally isometric to M_k^n.
In other words, for every point x ∈ X there is an ε> 0 and an isometry φ from B (x, ε) onto a ball B (φ (x), ε) ⊂ M_k^n.

Theorem (4.3): Let X be a complete, connected, n-dimensional manifold of constant curvature κ. When endowed with the induced length metric the universal covering of X is isometric to M_k^n.

Proof

The following proof is due to C. Ehresman. In the fi rst part of the proof we do not assume that X is complete. By definition, a chart φ : u → M_k^n is an isometry from an open set U ⊆ X onto an open set φ(u) ⊆ M $^n{}_κ$ .If φ : u → M $^n{}_κ$ is another chart and if u ∩ $\bar{u}$ is connected, then there is a unique isometry g ∈ Isom(M $^n{}_κ$) such that φ and g ∘ φ are equal on u ∩ $\bar{u}$. Consider the set of all pairs (φ, x), where φ : u → M ^{n}κ is a chart and x ∈ u .We say that two such pairs (φ, x) and (φ, x) are equivalent if x = $\dot{x}$ and if the restrictions of φ and $\acute{\varphi}$ to a small neighbourhood of x coincide. This is indeed an equivalence relation and the equivalence class of (φ, x) is called the germ of φ at X. Let ˆ X be the set of all equivalence classes, ie. the set.[12] pp45to46.

Lemma (4.4): Let (μ, g) be as above. Then for any p ∈ μ, p has no conjugate point along any geodesic emanating from p.

Proof:

Let γ be a geodesic emanating from p and X be a normal Jacobi field along γ with X(0) = 0. Let *f(t) = (X(t), X(t)).*Then $f^{/}$ *(t) = 2(∇γ˙ ∇γ X, X)*

and thus

$f^{//}$*(t) = 2(∇γ˙ ∇γ˙ X, X) + 2| ∇γ˙ X| 2*
= −2R(˙γ, X, γ, X˙) + 2| ∇γ˙ X| 2 ≥ 0.

Since $f^{/}$ (0) = 0, we must have f (t) ≥ 0 for all t > 0, i.e. f is non-decreasing. But we also know that for t small enough, f(t) > 0 since the zeroes of a Jacobi field is discrete. If follows that f(t) > 0 for all t. In other words, X has no zero along γ. So p has no conjugate point along γ.

Proof of the Cartan- Hadamard Theorem. According to lemma(6.1.5),

Exp $_p$: T $_p$ $\mu \to \mu$

is a local diffeomorphism everywhere. Let $\bar{g}$ = (exp $_p$) ∗ g, then $\bar{g}$ is a Riemannian metric on T $_p$ μ such that

exp $_p$: (T $_p$ μ,$\bar{g}$) → (μ, g)

is a local isometry. Note that the geodesics in (T $_p$$\mu$, $\bar{g}$) passing 0 ∈ T $_p$ μ are exactly the straight lines passing 0, which are defined for all t. It follows that $\exp_0$ is defined for all X_0 ∈ T_0(T $_p$ μ). According to the Hopf-Rinow theorem, (T $_p$ μ, $\bar{g}$) is complete. It follows from Ambrose Theorem that exp $_p$: T $_p$ μ → μ is a covering map.

If μ is simply connected, then any covering map to μ must be a global homeomorphism. Since $\exp_p$ is also a local diffeomorphism, it must be global diffeomorphism.

Example(4.5): Let μ be a Riemannian manifold of non-positive sectional curvature, I, e. K(∏) ≤ 0 For any 2-plane ∏ ⊂ TM .

(a) Let c : $[a, b]$ → M be a geodesic and let J be a Jacobi field along c. Let f (t) =$\|J(t)\|^2$. Show that $f^{//}$(t) ≥ 0, I .e ., f is a convex function.

(b) Derive from (a) that M does not a dmit conjugate points .

Solution

$f^{/}$ (t) = $\frac{d}{dt}\Big|_{t=0} \langle J(t), J(t)\rangle = 2\ \langle \frac{D}{dt} J(t), J(t)\rangle$

and

$f^{//}$(t) = 2($\langle \frac{D^2}{dt^2}\ J(t), J(t) + \ \left\|\frac{D}{dt} J(t)\right\|^2 \rangle$)

Using Jacobi equation, we conclude

$f^{//}$(t) = 2(- $\langle R(J(t), c^{\backslash}(t) c^{\backslash}(t), J(t)\rangle$ +$\left\|\frac{D}{dt} J(t)\right\|^2$)

We have $\langle R\left(J(t), c^{\backslash}(t)\right) c^{\backslash}(t), J(t)\rangle$ = 0 if J(t), $c^{\backslash}(t)$ are linear dependent and, otherwise, for

∏ = span (J(t), $c^{\backslash}(t) \subset T_{c(t)}M)$.

$\langle R\left(J(t), c^{\backslash}(t)\right) c^{\backslash}(t), J(t)\rangle$ = K (∏)($\|J(t)\|^2 \left\|c^{\backslash}(t)\right\|^2$-($\langle J(t), c^{\backslash}(t)\rangle^2$) ≤ 0,

Since sectional curvature is non-positive. This shows that $f^{//}(t)$, as a sum of two non-negative term, is greater than or equal to zero. (b) If there were a conjugate point q = c(t_2) to point p = c(t_1) along the geodesic c, then we would have a non –vanishing Jacobi field J along c with J(t_1) = 0 and J(t_2) = 0. This would imply that the convex, non – negative function f (t) = $\|J(t)\|^2$ would have zeros at t = t_1 and = t_2. This would force *f* to vanish identically on the interval $|t_1, t_2|$, which would imply that J vanishes as well, which leads to a contradiction [3] .pp 1to 2.

5. Matlab Solution

Algorithm1

% Let ? be a Riemannian manifold of non-positive sectional curvature, I, e. K(?) ? 0 For any 2-plane ? ? TM .

clear all
clc
syms t0 tl t d D f J f1 f2 d dt D(d,x) J1 J2 y x c(t) K(n) R(x,y) span span(x,y) n j(t) z(x,y) Z

% Let c : [a,b] ? M be a geodesic and let J be a Jacobi field along c.
t=1
f(t)= (abs(t))^2
f_1= diff(f(t))
f_2=diff(diff(f(t)))

% Since f^/ (0) = 0, we must have f (t) ? 0 for all t > 0, i.e. f is non-decreasing. But
% we also know that for t small enough, f(t) > 0 since the zeroes of a Jacobi field is discrete.
% If follows that f(t) > 0 for all t. In other words, X has no zero along ?.

x=j(t)
y=c(t)
n=span(x,y)
d=R(x,y)*y

J_1 = D(d,x)
Z=z(x,y)
J_2 = (K(n) * (abs(x))^2 * (abs(y))^2) - Z^2

The result

t =
1

f =
1

f_1 =
0

f_2 =
0

x =
j(1)

y =
c(1)

n =
span(j(1), c(1))

d =
c(1)*R(j(1), c(1))

J_1 =
D(c(1)*R(j(1), c(1)), j(1))

Z =
z(j(1), c(1))

J_2 =
K(span(j(1), c(1)))*abs(c(1))^2*abs(j(1))^2 - z(j(1), c(1))^2

Lemma (5.1): Assume that the sectional curvature of (N, h) is non- positive, and φ: (M, g) →(M, h) is abiharmonic mapping. then, it hold that $\Delta\,|T(\varphi)|^2 \geq 2\left|\nabla\,\ddot{T}(\varphi)\right|^2$

In M .Here, $\Delta = \sum_{i=1}^{m}(e_i^2 - \Delta_{ei} e_i)$ is the Laplace –Beltrami operator of (M, g).

Proof :

Let us take a local orthonormal frame field $(e_i)i^m$= 1 on M, and φ : (M, g) → (N, h), a biharmonic map, Then, for $V = T(\varphi) \in \Gamma(\varphi^{-1}TN)$,we have

$$\frac{1}{2}\,\Delta\,|\nu|^2 = \frac{1}{2}\sum_{i=1}^{m}(e_i^2\,|\nu|^2 - \Delta_{ei}\,e_i|\nu|^2)$$

$$= \sum_{i=1}^{m}(e_i h(\overline{\nabla}_{ei}\nu,\nu) - h(\overline{\nabla}_{\nabla ei}e_i\nu,\nu))$$

$$\sum_{i=1}^{m}(h(\overline{\nabla}_{ei}\overline{\nabla}_{ei}\nu\,,\nu) - h(\overline{\nabla}_{\nabla ei}\nu,\nu))$$

$$+\ \sum_{i=1}^{m} h(\overline{\nabla}_{ei}\,\nu,\overline{\nabla}_{ei}\nu)$$

$$=h(-\bar{\bar{\Delta}}\nu,\nu) + |\overline{\nabla}\nu|^2$$

$$=h(-R(\nu),\nu) + |\overline{\nabla}\nu|^2 \geq |\overline{\nabla}\nu|^2$$

Because for the second last equality, we used $\bar{\bar{\Delta}}\nu$ – R(ν) = J(ν) = 0 for ν =T(φ), due to the biharmonicity of φ : (M, g) → (N, h), and for the last inequality, we used

$$H(R(\nu),\nu) = \sum_{i=1}^{m} h(R^N(\nu,\varphi\,,e_i,\nu) \leq 0$$

Since the sectional curvature of (N, h) is non- positive.[11] pp13to34.

Matlab Solution (5.2):

Algorithm2

```
% Let (?, g) be as above. Then for any p ? ?, p has no conjugate point along any geodesic emanating from p
clear all
clc
syms deltav v m i e s f1 f2 f f(x,y) t h(deltav,v) f3 R(v)

% Let us take a local orthonormal frame field (e_i)i^m= 1 on M, and ? : (M, g) ? (N, h), a biharmonic map, Then, for V=T(?)?? (?^(-1) TN),we have

f1 = 1/2 * (abs(v))^2
f2 = h(deltav,v) + (abs(deltav))^2
f3 = h(-R(v),v) + (abs(deltav))^2

% Because for the second last equality, we used ? ?? – R(?) = J(?) = 0 for ? =T(?), due to the biharmonicity of ? : (M, g) ? (N, h), and for the last inequality, we used
% H(R(?),?) = ?_(i=1)^m??h(R^N (?, ??,e_i,?) ?0
% Since the sectional curvature of (N, h) is non- positive
J(v) = deltav - R(v) == 0
```

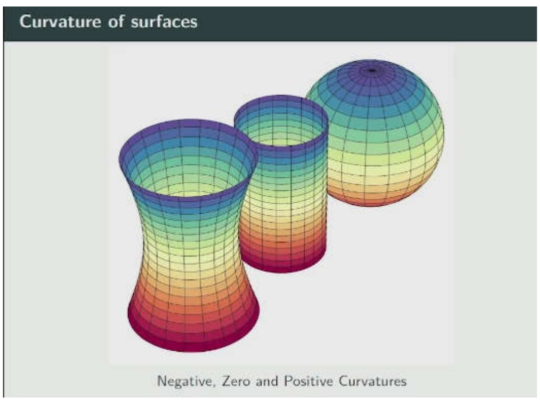
The result

f_1 =
abs(v)^2/2

f_2 =
abs(deltav)^2 + h(deltav, v)

f_3 =
h(-R(v), v) + abs(deltav)^2

J(v) =
deltav - R(v) == 0



6. RESULTS

After we showed the calculation of Non –Positive Sectional Curvature using Matlab we found the following some results :
We showed that Matlab gives precise results of high speed compared with that of Manual, also we stated the ability capability of Graphs or Diagram drawing to any curvature via Matlab, we explained the possibility of the calculation of Non –Positive Sectional Curvature by Matlab with a very high rate and accuracy finally we shed lights on as Cartan –Hadamard theory which is considered one of the most important applications regarding Non –Positive sectional curvature.

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