



APPLICATION AND COMPUTER REVIEW OF SOME PROBABILITY MODELS

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Abstract

The Poisson random variable has a tremendous range of application in diverse areas, because it may be used as an approximation for a binomial random variable with parameters (n,p). In practice, the exponential distribution often arises as the distribution of the amount of time until some form now until an earthquake occurs, or until a telephone call you receive turns out to be a wrong number at all random variables that tend in practice to have exponential distribution.

Keywords: Poisson model, probability density function, random variable, MATLAB, normal distribution.

INTRODUCTION

The concept of probability science has been invented at the beginning of seventeenth century. It started with scientific research on random games and lucky issues. The most famous mathematicians, Pascal and Ferma have laid the foundation of probability. The importance of probability science has increased in recent years since the concept was developed with the knowledge of Emarto and used in various fields of engineering [2, 5]. Continues random variable is used in the field of engineering, economics and business. Random transformation is a quantity that receives its values with probability, in other words, we call a quantity random when, as a result of a test, it takes different values that are not known before the test and are denoted by x. For example, in tossing coins, we are only interested in knowing the numbers of Head (H) or Tail (T). Therefore, the random variable x represents the number of Head (H) or Tail (T). Random variable x is continuous random variable which takes on values in a continuous set. In general, quantities such as pressure, height, mass weight, density, volume, temperature and distance are examples of continuous random variables. Continuous random variables are represented by numerical values along a continuous line. The amount of continuous random variable depends on the result of values. For example, if the values are related to the size of the weight of students, the weight of students will be a continuous random variable. Also, if the values are to determine the time interval between customers entering the hotel, the time interval will be the continuous random variable. In both cases, the value of the random variable depends on the result of the statistical flow. For example, a person who randomly chooses his weight after measuring 80 kg, will have random change of 80kg. Similarly, if the time interval between customers entering the hotel is 3 minutes, the value of our random change will be 3 minutes. The probability of randomly selected people have weight between a low and high limit, or the probability of randomly selected people have weight between a low and high limit, or the probability of customers entering certain time interval, is determined by definite integral. The function from which the integral is derived is called the probability density function. The probability of an event is a number describing the chance that the event will happen between zero and 1.

If the chance of being an event was 0 this would mean it would never happen and probability of 1 means that the event will happen.

Research goal

The goal of research is application and computer review of some probability models, which is difficult to calculate without a computer. As a result, the study of different programs has been written in the language of MATLAB programming [1].

Research method

The research methods in this work are computer and library methods and to achieve this goal, we have been used scientific sources and internet websites.

Main results (Research topic)

If f(x) is a function of the probability density function, then according to definition we have:

∫x f(x)dx = 1.

Where the integral limits include all possible values of random variable x. We know that probability cannot be negative, so the probability density function must be positive. That means: f(x) > 0.

For the result or income of an event, we have a random test that results in a random variable is between a and b. We have:

∫a b f(x)dx.

And finally, the probability that the weight of randomly selected people will be at a certain distance, will be obtained by integrating the density function at that distance.

In the figure below, the probability that an individual weighs between 140 and 180 kg is determined by:

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$$\int_{140}^{180} f(x) dx.$$

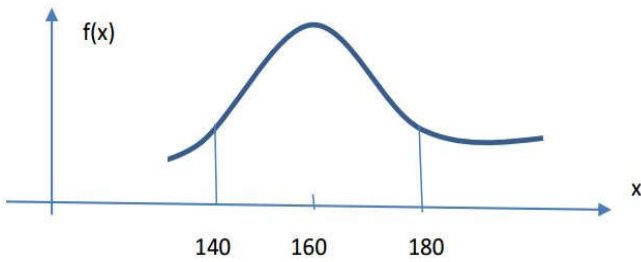


Figure 1. the range of function between 140 and 180

Now we want to explain the concept of probability as a definite integral value by introducing some important probability models. One of these functions is the probability density function of uniform or rectangular distribution [4].

**Uniform distribution**

A uniform distribution of the experiment that randomly selects the value of x in the region as  $a \leq x \leq b$ . All values of x in the interval have equal probability and their density function is equal to:

$$f(x) = \frac{1}{b-a}, a \leq x \leq b.$$

The density function is zero for the value of x outside the interval. This function is shown in the figure below.

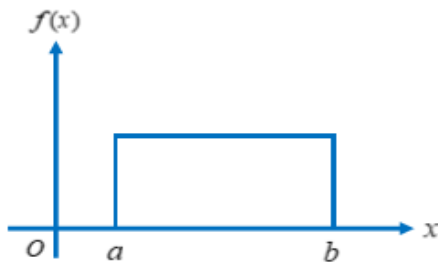


Figure 2. Density function for x values

If density function is:  $f(x) = \frac{1}{b-a}$  for  $a \leq x \leq b$ .

Definite integration for  $f(x)$  from  $a$  to  $b$  is equal to:

$$\int_a^b \frac{1}{b-a} dx = \frac{x}{b-a} \Big|_a^b = \frac{b-a}{b-a} = 1.$$

Example 1: The bus of some direction move after every half hour. What is the probability that a person who has come to the bus stop by accident to wait for at least 20 minutes?

Solution:

The random variable t, which has reached the waiting time, is the next service and  $0 \leq t \leq 30$  distributed normally. The probability that the person will wait at least 20 minutes is equal to:

$$P(t \geq 20) = \int_{20}^{30} \frac{1}{30} dt = \frac{1}{3}$$

Example 2: Students' points in a math class are uniformly distributed between 12 and 20. Determine:

- a) Probability distribution function
- b) The probability of the points being (13-15), (18-20)
- c) Average points of class, variance

Solution:

$$f_x(x) = \begin{cases} \frac{1}{20-12} = \frac{1}{8}, & 12 \leq X \leq 20 \\ 0, & \text{for all value.} \end{cases}$$

$$P(18 \leq X \leq 20) = \int_{18}^{20} \frac{1}{8} dx = \frac{1}{8}(20-18) = \frac{1}{4}$$

$$P(13 \leq X \leq 15) = \int_{13}^{15} \frac{1}{8} dx = \frac{1}{8}(15-13) = \frac{2}{8} = \frac{1}{4}$$

$$E[x] = \frac{20+12}{2} = 16$$

$$var(X) = \frac{(20-12)^2}{12} = \frac{64}{12} = \frac{16}{3}.$$

**Model of Exponential Density Function**

One of the uses of this model is for a person to enter a station. For example; the time between customers entering a station is often described as a probability distribution [6, 8]. The exponential probability density function is defined by:

$$f(t) = ke^{-kt}, 0 \leq t < \infty.$$

Where, this time between entries,  $e = 2.71828$  constant number and k is the average number of people entered over a period of time. This distribution is shown in the figure below. The probability of a person entering in the time interval  $t \leq T$  is in the following manner:

$$p(t \leq T) = \int_0^T ke^{-kt} dt$$

The exponential density function provides the necessary conditions for a density function, since  $f(t) \geq 0$  then:

$$\int_0^{\infty} ke^{-kt} dt = -e^{-kt} \Big|_0^{\infty} = 0 - (-1) = 1$$

Example 3: Customers enter a station to be distributed exponentially with an average of  $k = 10$  per hour. What is the probability that an arrival will have to enter in  $T = 0,2$  hour.

Solution:

$$p(t \leq 0.2) = \int_0^{0.2} 10e^{-10t} dt = -e^{-10t} \Big|_0^{0.2} = -e^{-2} + 1 = -0.135 + 1 = 0.865$$

Example 4: It has been observed that in an airport, airplanes land on Earth at an average of  $k = 20$  per hour according to the exponential distribution. What is the probability of an airplane arriving in  $(1/20$  hour) 3 minutes?

Solution:

$$p(t \leq 1/20 \text{ hr}) = \int_0^{1/20} 20e^{-20t} dt = -e^{-20t} \Big|_0^{1/20} = -e^{-1} + 1 = 0.632$$

Now we introduce briefly computer review of this topic.

### Normal model (Normal Distribution):

Random variable  $x$  has a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . And denoted by  $X \sim N(\mu, \sigma^2)$  if the density function is as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < +\infty, \quad \sigma^2 \in (0, +\infty)$$

Normal distribution is drawn with the **norm plot** command in MATLAB.

The question arises that why we prefer the graph of normal probability [5; 7]. In answer to the question, we must say that the normal probability plot is a graphical technique for assessing whether or not a data set is approximately normally distributed and the normal probability plot is a special case of the probability plot. We cover the normal probability plot separately due to importance in many applications [1, 2]. Consider the following example.

```
x = normrnd(10,1,25,11); normplot(x)
```

It should be noted that the Normand command in MATLAB program is used to generate random numbers which has normal probability.

The command *Normand(mu, sigma, n, m)* generates normal random numbers which have normal probability with mean  $\mu$ , and standard deviation  $\sigma$  and the scalars  $m$  and  $n$  are the row and column of a matrix.

### RESULT

```
mu = 0; sigma = 1; pd = makedist('normal', mu, sigma);
```

Now if we consider vector  $x = [-2 -1 0 1 2]$ . In this case, pdf values for standard normal distribution for the mentioned values are received as  $y = pdf(pd, x)$ .

### Results and suggestions

After reviewing and researching, the above topic is concluded in the following:

1. In statistics and probability some models are discontinuous, which describes the probability that an event will happen in specified interval of times or at a fixed location, while these events happen with specific and independent average value from the time of the last event.
2. The Poisson model is used for a number of events in specific interval, such as distance, area and volume.
3. The Poisson distribution predicts the degree of dispersion around the average value of the event.
4. Distribution can be considered as an approximation of binomial distribution.

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