



## FORECASTING USING FUZZY TIME SERIES INVARIANT MODEL

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### Abstract

Forecasting shows a dynamic part in numerous fields such as, rainfall forecasting, stock market forecasting, weather forecasting and so on. In recent years, fuzzy time series is used for forecasting the time series data. Song and Chissom (1993) proposed fuzzy time series for forecasting the enrollments of University of Alabama. Shiva Raj Singh(2007) presented a simple time variant method for forecasting the enrollments of University of Alabama. In the present work, a modified fuzzy time series algorithm is used for such type of problems. The forecasting results are better than the existing methods. Mean Square Error (MSE) is minimum when comparing the existing methods. The results were displayed numerically and graphically.

**Keywords:** Fuzzy Time Series, Mean Square Error, Forecasting, Line Graph, Bar Diagram and Enrollments.

### INTRODUCTION

The time series is a sequence of values arranged in a specific order of time. Forecasting predicts the future values of the time series. For the past decades, fuzzy time series has been commonly used for forecasting the chronological data. Fuzzy time series is used by arrangement with forecasting problems in which the past data are linguistic values. Song and Chissom (1993) proposed the fuzzy time series for the enrollments of the university of Alabama. Chen and Hsu (2004) presented a method to forecast the fuzzy time series on the same problem. Song Chissom(1994) offered a first order time variant and a time invariant model for forecasting the enrollments of University of Alabama. Chen (2002) existing a forecasting method based on high-order fuzzy time series for forecasting enrollments of the University of Alabama. Own and Yu (2005) offered a heuristic higher order model to overcome the ambiguity in trend raised by Chen with introducing a heuristic function. Tahseen Ahmed Jilani *et al.* (2007) proposed frequency density based partitioning of the historical data based on  $k^{\text{th}}$  order time-variant model. Chen and Hwang (2000) offered a method based on fuzzy time series to forecast the daily temperature. Jilani *et al.* (2007) presented new fuzzy time metrics for high order multivariate fuzzy time series forecasting for car road accidents casualties in Belgium. Singh (2007) overwhelms the deficiency of Chen's high-order model of the fuzzy time series. Minimum-maximum composition operations are used for fuzzy relation. Meredith Stevenson *et al.* (2009) presented the fuzzy time series for the enrollments of university of Alabama. Arumugam (2010) discussed a modified fuzzy time series models for forecasting the Enrollments University of Alabama. Viswam and Satyanarayana Reddy(2018) predicted short term share market using Autoregressive Integrated Moving Average (ARIMA) model. Wanie *et al.* (2020) predicted rainfall data in Tasik Kenyir using neural networks. Moulana *et al.* (2020) predicted short term and long term rainfall using machine learning methods. In this work, a modified fuzzy time series algorithm is used when comparing the existing methods, in Mean Square Error is minimized.

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### Computational algorithm for fuzzy time series

Fuzzy time series used to handle forecasting problems. It gives better forecasting accuracy when comparing the existing methods using computational algorithm. Chen (2002), Chen and Chia-Ching Hsu (2004), Sun Xihao and Liyimin (2008), and Arumugam (2010). The step by step procedure is as follows:

**Step1:** Compute the first order variation of the historical data.

**Step2:** Define the universe of discourse, U based on the range of available historical data.

$$U=[D_{\min}-D_1, D_{\max}+D_2]$$

Where  $D_{\min}$  is the minimum value of the first order variation of the historical data,

$D_{\max}$  is the maximum value of the historical data and  $D_1, D_2$  are two positive integers.

**Step 3:** Partition the universe of discourse U into equal length intervals:  $u_1, u_2, \dots, u_m$ .

**Step 4:** The number of intervals will be in agreement with the number of linguistic variables (fuzzy sets)  $A_1, A_2, \dots, A_m$  to be considered.

**Step 5:** Fuzzify the variations of the historical data and established the fuzzy logical relationships represented by  $A_i, A_j$ .

**Step 6:** Rules for forecasting:

$[A_j]$  is corresponding interval  $u_j$  for which membership in  $A_j$  is supremum (i.e., 1)

$L[A_j]$  is the highest value of the interval  $u_j$  having supremum value in  $A_j$ .

$M[A_j]$  is the mid value of the interval  $u_j$  having supremum value in  $A_j$ .

For a fuzzy logical relationship  $A_i \rightarrow A_j$ .

- $A_i$  is the fuzzified enrollments of the current year  $n$ ;
- $A_j$  is the fuzzified enrollments of the next year  $n+1$ ;
- $D_i$  is the actual enrollment of the current year  $n$ ;
- $D_{i-1}$  is the actual enrollment of the previous year  $n-1$ ;
- $E_i$  is the variation enrollment of the current year  $n$ ;
- $E_{i-1}$  is the variation enrollment of the previous year  $n-1$ ;
- $F_j$  is the forecasted enrollments of the next year  $n+1$ ;

**Step 7:** Forecasting enrollments for the year  $n+1$  is obtained from modified computational algorithm as follows;

Obtain the fuzzy logical relationship  $A_i \rightarrow A_j$ .

If  $E_i < M[A_j]$ , then  $F_j = D_{i-1} + (M[A_j] - 1/6L[A_j])$   
 else If  $E_i > M[A_j]$ , then  $F_j = D_{i-1} + (M[A_j] + 1/6L[A_j])$   
 else  $F_j = D_{i-1} + M[A_j]$

**Step 8:** Obtain the mean square error using actual values and forecasted values

$$MSE = \frac{\sum_{i=1}^n (Actual\ Value - Forecasted\ Value)^2}{n}$$

**RESULTS AND DISCUSSION**

**Step 1:** Compute the first order difference of the chronological data.

**Step 2:** The universe of discourse  $U$  is defined as

$$U = [D_{min} - D_1, D_{max} + D_2]$$

$$U = [-955 - 45, 1291 + 109] = [-1000, 1400]$$

Where  $D_{min} = -955$  is the minimum value of the first order difference of the chronological data.

$D_{max} = 1291$  is the maximum value of the first order difference of the chronological data,

$D_1 = 45$  and  $D_2 = 109$  are two positive integers.  $D_1, D_2$  are selecting intuitively for the rounded off  $U$  value.

**Step 3:** The universe of discourse  $U$  is isolated into eight equal length of interval.

$$U_1 = [-1000, -700] \quad U_2 = [-700, -400] \quad U_3 = [-400, -100]$$

$$U_4 = [-100, 200]$$

$$U_5 = [200, 500] \quad U_6 = [500, 800] \quad U_7 = [800, 1100]$$

$$U_8 = [1100, 1400]$$

**Step 4:** Define eight fuzzy sets  $A_1, A_2, \dots, A_8$  taking some linguistic values on the creation of discourse  $U$ . The linguistic values are as follows:

- $A_1$ =very very low;  $A_2$ =very low;  $A_3$ = low;
- $A_4$ = medium normal;  $A_5$ = normal;  $A_6$ = high;
- $A_7$ = very high;  $A_8$ = very very high

**Table 1. Fuzzified Enrollment Data on Variations**

Years	Actual enrollments	Variations	Fuzzified variations
1971	13055	-	
1972	13563	508	$A_6$
1973	13867	304	$A_5$
1974	14696	829	$A_7$
1975	15460	764	$A_7$
1976	15311	-149	$A_3$
1977	15603	292	$A_5$
1978	15861	258	$A_5$
1979	16807	946	$A_7$
1980	16919	112	$A_4$
1981	16388	-531	$A_2$
1982	15433	-955	$A_1$
1983	15497	64	$A_4$
1984	15145	-352	$A_3$
1985	15163	18	$A_4$
1986	15984	821	$A_7$
1987	16859	875	$A_7$
1988	18150	1291	$A_8$
1989	18970	820	$A_7$
1990	19328	358	$A_5$
1991	19337	9	$A_4$
1992	18876	-461	$A_2$

**Step 5:** The membership of above revealed linguistic variables are assigned through the trapezoidal membership function by fixing the values randomly. The memberships of the linguistic variables are as follows.

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8$$

$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8$$

$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8$$

$$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7 + 0/u_8$$

$$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7 + 0/u_8$$

$$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_7 + 0/u_8$$

$$A_7 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7 + 0.5/u_8$$

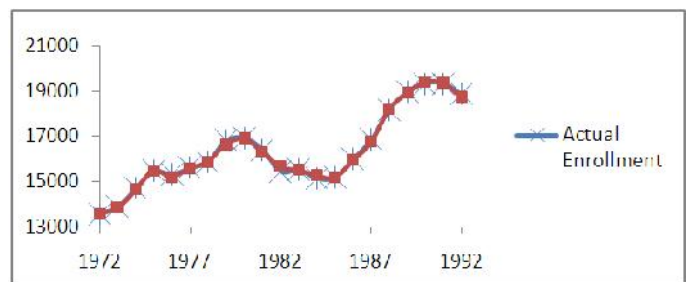
$$A_8 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0.5/u_7 + 1/u_8$$

**Step 6:** The historic variations of the time series data are fuzzified in order to have the fuzzy logical relations attained as follows:

Variations in the fuzzy logic relationships

- $A_6 \rightarrow A_5, A_5 \rightarrow A_7, A_7 \rightarrow A_7, A_7 \rightarrow A_7, A_3 \rightarrow A_3, A_5 \rightarrow A_5, A_5 \rightarrow A_5$
- $A_5 \rightarrow A_7, A_7 \rightarrow A_4, A_4 \rightarrow A_4, A_2 \rightarrow A_2, A_1 \rightarrow A_1, A_4 \rightarrow A_4, A_3 \rightarrow A_3$
- $A_3 \rightarrow A_4, A_4 \rightarrow A_4, A_4 \rightarrow A_4, A_7 \rightarrow A_7, A_8 \rightarrow A_8, A_7 \rightarrow A_5, A_5 \rightarrow A_5$
- $A_5 \rightarrow A_4, A_4 \rightarrow A_2$

**Step-7:** The forecasted values have been achieved by means of computational algorithm. Then the forecasted output while comparing with different methods given in table 2.



**Figure1 Actual and Forecasted Enrollments of University of Alabama**

Figure 1 shows that actual and forecasted enrollments of university of Alabama

Table 2. Forecasted Enrollments by Different Models

Year	Actual Enrollments	Tsaur et al. (2005)	Lee and Chou(2004)	Chen (2002) order 3	S. R. Singh (2007) order3	P.Arumugam (2010)	Proposed
1971	13055	---	---	---	---	---	---
1972	13563	14,000	14,025	---	---	13705	13572
1973	13867	14,000	14,568	---	---	13863	13830
1974	14696	15,500	14,568	14500	14750	14817	14634
1975	15460	16,000	15,654	15500	15750	15646	15479
1976	15311	16,000	15,654	15500	15500	15610	15193
1977	15603	16,000	15,654	15500	15500	15611	15578
1978	15861	16,000	15,654	15500	15500	15953	15870
1979	16807	16,833	16,197	16500	16500	16811	16628
1980	16919	16,833	17,283	16500	16500	16857	16890
1981	16388	16,833	17,283	16500	16500	16369	16302
1982	15433	16,833	16,197	15500	15500	15538	15655
1983	15497	16,000	15,654	15500	15500	15483	15516
1984	15145	16,000	15,654	15500	15250	15247	15264
1985	15163	16,000	15,654	15500	15500	15195	15162
1986	15984	16,000	15,654	15500	15500	15213	15930
1987	16859	16,000	16,197	16500	16500	16934	16751
1988	18150	16,833	17,283	18500	18500	18109	18192
1989	18970	19,000	18,369	18500	18500	19100	18917
1990	19328	19,000	19,454	19500	19500	19120	19403
1991	19337	19,000	19,454	19500	19500	19378	19345
1992	18876	---	---	18500	18750	18787	18720

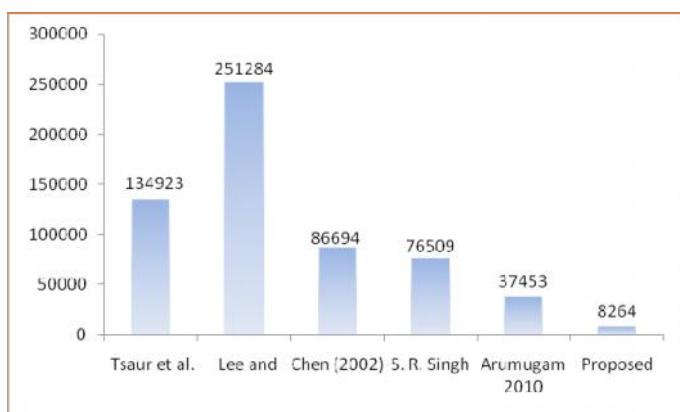


Figure 2. Mean Square Error Values of Enrollments of the University of Alabama

Figure 2 shows that mean square error is minimum when compared to other models.

## Conclusion

In this paper, a modified fuzzy time series algorithm is used for forecasting the enrollments of University of Alabama. It provides higher accuracy when comparing Chen's (2002) time invariant model, Singh (2007) time-variant model and Arumugam (2010) time-variant model. It gives minimum mean square error when comparing the existing methods. Mean square error value is 8264.

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