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Research Article

MATRICES: CONCEPTS AND REAL WORLD CONTEXTS

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Abstract

The very mention of "matrices" almost compels people to make a mental about turn. Most of us are driven instinctively to run for the hills, leaving matrices for the "experts", the people deeply involved in mathematics and the sciences. This paper seeks to demystify the concepts of matrices as well as to identify some of the many ways in which we use matrices in our everyday lives. The first section of this paper attempts to show, in very simple terms, how matrices work. The second section of this paper demonstrates some of the many ways in which we use matrices, often without being conscious of the fact that we are using them to initiate, develop, and sustain our enterprises.

Keywords: Classroom Instruction, Outcomes, High schools, Teaching and Learning, Monitoring.

INTRODUCTION

Defining a Matrix

A matrix is a set of elements, organized into rows and columns. These elements may consist of either numbers or variables. A matrix may also contain fractions and/or decimals. The order of a matrix is the number of rows multiplied by the number of columns. m x n matrix is a matrix of m rows and n columns with an order (m x n).

Thus A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a *2 x 2 matrix* since it consists of 2 rows and 2 columns.

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{matrix} R1 \\ R2 \end{matrix}$
A = $C1 \quad C2$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Therefore m = 2 since a row consists of all elements in the same horizontal line. And n = 2 since a column consists of all elements in the same vertical line.

The matrix B = [a] is a *1 x 1 matrix* since it consists of one row and one column.

The matrix C = $\begin{bmatrix} a \\ b \end{bmatrix}$ is a *2 x 1 matrix* and is called a *column matrix* (vertical array of elements).

The matrix D = [a b] is a 1 x 2 matrix and is called a *row matrix* (horizontal array of elements).

The matrix E = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2 x 2 matrix and is called a *square matrix* (same number of rows as columns).

The matrix F = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called a *null or a zero matrix* (all elements are zero). The null matrix or zero matrix is the identity matrix for the addition of 2 x 2 matrices.

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The matrix G = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ is called a *diagonal matrix* (all the elements except the leading diagonal elements are zero).

The matrix I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called a *unit matrix* (each leading diagonal element is 1) and is denoted by the letter *I* and has a magnitude of 1. The unit matrix is the identity for the multiplication of 2 x 2 matrices.

Addition of Matrices

Matrices of the same order can be added or subtracted by adding or subtracting corresponding elements.

Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$and C = $\begin{bmatrix} j & k \\ l & m \end{bmatrix}$

Find (i) A + B
(ii) B + A
(iii) B + C
(iv)A + B + C

(i) A + B = $\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

(ii) B + A = $\begin{bmatrix} e+a & f+b \\ g+c & h+d \end{bmatrix}$

(iii) B + C = $\begin{bmatrix} e+j & f+k \\ g+l & h+m \end{bmatrix}$

(iv) (A + B) + C = $\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} + \begin{bmatrix} j & k \\ l & m \end{bmatrix}$
= $\begin{bmatrix} a+e+j & b+f+k \\ c+g+l & d+h+m \end{bmatrix}$

OR

A + (B + C) = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e+j & f+k \\ g+l & h+m \end{bmatrix}$
= $\begin{bmatrix} a+e+j & b+f+k \\ c+g+l & d+h+m \end{bmatrix}$

Example:

If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 3 \\ 7 & 3 \end{bmatrix}$and C = $\begin{bmatrix} 11 & 6 \\ 13 & 17 \end{bmatrix}$

Find (i) A + B
(ii) B + A
(iii) B + C
(iv) A + B + C

(i) A + B = $\begin{bmatrix} 1+5 & 2+3 \\ 3+7 & 4+3 \end{bmatrix}$

A + B = $\begin{bmatrix} 6 & 5 \\ 10 & 7 \end{bmatrix}$

(ii) B + A = $\begin{bmatrix} 5+1 & 3+2 \\ 7+3 & 3+4 \end{bmatrix}$

B + A = $\begin{bmatrix} 6 & 5 \\ 10 & 7 \end{bmatrix}$

The addition of matrices is commutative.

(iii) B + C = $\begin{bmatrix} 5+11 & 3+6 \\ 7+13 & 3+17 \end{bmatrix}$
B + C = $\begin{bmatrix} 16 & 9 \\ 20 & 20 \end{bmatrix}$

(iv) A + B + C = (A + B) + C

= $\begin{bmatrix} 6 & 5 \\ 10 & 7 \end{bmatrix}$ + $\begin{bmatrix} 11 & 6 \\ 13 & 17 \end{bmatrix}$

= $\begin{bmatrix} 17 & 11 \\ 23 & 24 \end{bmatrix}$

OR
A + B + C = A + (B + C)

= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ + $\begin{bmatrix} 16 & 9 \\ 20 & 20 \end{bmatrix}$

= $\begin{bmatrix} 17 & 11 \\ 23 & 24 \end{bmatrix}$

Therefore A + B + C = (A + B) + C = A + (B + C)

The addition of matrices is associative.

Subtraction of Matrices

Let A = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and B = $\begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix}$

Find (i)A – B
(ii) B - A
(i) A – B = $\begin{bmatrix} a-g & b-h & c-i \\ d-j & e-k & f-l \end{bmatrix}$
(ii) B – A = $\begin{bmatrix} g-a & h-b & i-c \\ j-d & k-e & l-f \end{bmatrix}$

Example:

If A = $\begin{bmatrix} 7 & 3 & 12 \\ 6 & 5 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 9 & 4 \\ 11 & 6 & 5 \end{bmatrix}$

Find (i)A – B
(ii) B - A

(i) A – B = $\begin{bmatrix} 7-3 & 3-9 & 12-4 \\ 6-11 & 5-6 & 0-5 \end{bmatrix}$
A – B = $\begin{bmatrix} 4 & -6 & 8 \\ -5 & -1 & -5 \end{bmatrix}$

(ii) B – A = $\begin{bmatrix} 3-7 & 9-3 & 4-12 \\ 11-6 & 6-5 & 5-0 \end{bmatrix}$

B – A = $\begin{bmatrix} -4 & 6 & -8 \\ 5 & 1 & 5 \end{bmatrix}$

The subtraction of matrices is non-commutative. That is, A – B ≠ B – A

Scalar Multiplication

To perform scalar multiplication, multiply each element in the matrix by the scalar quantity or constant *(k)*.

Given A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then kA = $k\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

kA = $\begin{bmatrix} k\times a & k\times b \\ k\times c & k\times d \end{bmatrix}$

kA = $\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

Example:

1. Given that A = $\begin{bmatrix} 6 & 5 \\ 10 & 7 \end{bmatrix}$ and k = 3, determine kA

kA = 3$\begin{bmatrix} 6 & 5 \\ 10 & 7 \end{bmatrix}$

kA = $\begin{bmatrix} 3\times 6 & 3\times 5 \\ 3\times 10 & 3\times 7 \end{bmatrix}$

kA = $\begin{bmatrix} 18 & 15 \\ 30 & 21 \end{bmatrix}$

2. Given that B = $\begin{bmatrix} 7 & 3 & 12 \\ 6 & 5 & 0 \end{bmatrix}$ and k = -2, determine kB

kB = -2 $\begin{bmatrix} 7 & 3 & 12 \\ 6 & 5 & 0 \end{bmatrix}$

kB = -2$\begin{bmatrix} -2\times 7 & -2\times 3 & -2\times 12 \\ -2\times 6 & -2\times 5 & -2\times 0 \end{bmatrix}$

kB = $\begin{bmatrix} -14 & -6 & -24 \\ -12 & -10 & 0 \end{bmatrix}$

3. Given that C = $\begin{bmatrix} -3 & 0 \\ 4 & 5 \end{bmatrix}$and $k = \frac{1}{2}$, determine kC

$kC = \frac{1}{2}\begin{bmatrix} -3 & 0 \\ 4 & 5 \end{bmatrix}$

$kC = \begin{bmatrix} \frac{1}{2}\times -3 & \frac{1}{2}\times 0 \\ \frac{1}{2}\times 4 & \frac{1}{2}\times 5 \end{bmatrix}$

$$kC = \begin{bmatrix} \frac{-3}{2} & 0 \\ 2 & \frac{5}{2} \end{bmatrix}$$

Matrix Multiplication

For multiplication to be defined, the "inner" numbers must match. The result will be determined by the "outer" number. That is, if A is an m x n matrix and B is an n x m matrix, n is considered the "inner" number while m is considered the "outer" number. Therefore, n x n means the matrix is compatible for multiplication which will give a result of an m x m matrix. Multiplication is done by multiplying the rows of the first matrix by the columns of the second matrix.

Given that A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

A x B = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ x $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

A x B = $\begin{bmatrix} a \times e + b \times g & a \times f + b \times h \\ c \times e + d \times g & c \times f + d \times h \end{bmatrix}$

A x B = $\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$

Example:

1. Given A = $\begin{bmatrix} 3 & 4 \\ 7 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 9 & 7 \end{bmatrix}$, determine the value of AB

A = 2 x 2 matrix
B = 2 x 3 matrix

Therefore, A and B are compatible for multiplication since the number of columns in the first matrix (2) is equal to the number of rows in the second matrix (2). The resulting matrix should be equal to the number of rows in the first matrix (2) by the number of columns in the second matrix (3); a 2 x 3 matrix. Proving:

A x B = $\begin{bmatrix} 3 & 4 \\ 7 & 2 \end{bmatrix}$ x $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 9 & 7 \end{bmatrix}$

$$\begin{bmatrix} 3 \times 3 + 4 \times 6 & 3 \times 1 + 4 \times 9 & 3 \times 5 + 4 \times 7 \\ 7 \times 3 + 2 \times 6 & 7 \times 1 + 2 \times 9 & 7 \times 5 + 2 \times 7 \end{bmatrix}$$

$$\begin{bmatrix} 9 + 24 & 3 + 36 & 15 + 28 \\ 21 + 12 & 7 + 18 & 35 + 14 \end{bmatrix}$$

$$\begin{bmatrix} 33 & 39 & 43 \\ 33 & 25 & 49 \end{bmatrix}$$

The product of AB is a 2 x 3 matrix.

2. Find the product of the following pair of matrices:

$$\begin{array}{ccc} 2 & -1 & 0 \\ 0 & 5 & 3 \\ 1 & -2 & -1 \end{array} \quad \begin{array}{ccc} 1 & 0 & -1 \\ 2 & 3 & 3 \end{array}$$

3 x 3 2 x 3

The above matrices are *undefined* because the number of columns in the first matrix (3) is different from the number of rows in the second matrix (2). Therefore, multiplication cannot take place.

Square of a Matrix

The square of a matrix is obtained by multiplying the matrix by itself, as in the exponentiation of numbers (i.e. a^2 = a x a). The most basic requirement for matrix exponentiation to be *defined* is that the matrix must be 'square'.

Given that A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then A^2 = A x A

That is, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ x $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

A^2 = $\begin{bmatrix} a \times a + b \times c & a \times b + b \times d \\ c \times a + d \times c & c \times b + d \times d \end{bmatrix}$

A x B = $\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$

Example:

1. Given that A = $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, find the value of A^2

A^2 = A x A

= $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ x $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$

= $\begin{bmatrix} 4 \times 4 + (-2) \times 6 & 4 \times (-2) + (-2) \times (-3) \\ 6 \times 4 + (-3) \times 6 & 6 \times (-2) + (-3) \times (-3) \end{bmatrix}$

= $\begin{bmatrix} 16 + (-12) & (-8) + 6 \\ 24 + (-18) & (-12) + 9 \end{bmatrix}$

= $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$

2. Given that B = $\begin{bmatrix} 4 & -5 \\ 4 & -5 \end{bmatrix}$, evaluate B^2

B^2 = B x B

= $\begin{bmatrix} 4 & -5 \\ 4 & -5 \end{bmatrix}$ x $\begin{bmatrix} 4 & -5 \\ 4 & -5 \end{bmatrix}$

= $\begin{bmatrix} 4 \times 4 + (-5) \times 4 & 4 \times (-5) + (-5) \times (-5) \\ 4 \times 4 + (-5) \times 4 & 4 \times (-5) + (-5) \times (-5) \end{bmatrix}$

= $\begin{bmatrix} 16 + (-20) & (-20) + 25 \\ 16 + (-20) & (-20) + 25 \end{bmatrix}$

= $\begin{bmatrix} -4 & 5 \\ -4 & 5 \end{bmatrix}$

In example 2, the output matrix B^2 is the same as the original matrix *B*, except every element has been multiplied by -1. Hence, B^2 can be written in terms of itself by the expression $B^2 = -B$

3. Given that C = $\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$, evaluate C^2

C^2 = C x C

= $\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ x $\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$

= $\begin{bmatrix} 1 \times 1 + (-3) \times 2 & 1 \times (-3) + (-3) \times 5 \\ 2 \times 1 + 5 \times 2 & 2 \times (-3) + 5 \times 5 \end{bmatrix}$

= $\begin{bmatrix} 1 + (-6) & (-3) + (-15) \\ 2 + 10 & (-6) + 25 \end{bmatrix}$

= $\begin{bmatrix} -5 & -18 \\ 12 & 19 \end{bmatrix}$

Equal Matrices

Two matrices are equal if:

- They have the same dimensions, that is, rows by column (m x n).
- Every element of the first matrix is the same as every element of the second matrix in the corresponding positions.

Given that A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

Then A = B is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

With a = e, b = f, c = g, and d = h

Example:

1. If A and B are equal matrices, find the value of a, b, x, and y.

A = $\begin{bmatrix} a & 5 & -3 \\ 3 & b & 0 \end{bmatrix}$

B = $\begin{bmatrix} 1 & 5 & x \\ y & -7 & 0 \end{bmatrix}$

Equating corresponding elements: a = 1, b = -7, x = -3, y = 3

Hence, a = 1, b = -7, x = -3, y = 3

2. Given that $\begin{bmatrix} 4x+5 & 9 \\ 7 & -7 \end{bmatrix} = \begin{bmatrix} 21 & 9 \\ 7 & y-12 \end{bmatrix}$

Find the values of: (i) x
(ii) y

Equating corresponding elements: $4x + 5$ = 21
$y - 12$ = -7

Algebraically:

$4x + 5$ = 21
$x = \frac{21-5}{4}$
x = 4
$y - 12$ = -7
y = (-7) + 12
y = 5
Hence x = 4 and y = 5

Determinant of a 2 x 2 Matrix

The determinant of a matrix is the product of the non-leading diagonal elements subtracted from the product of the leading diagonal elements. The determinant of a matrix is denoted det(A), det A, or IAI. Determinants turn out to be very useful when studying more advanced topics such as *inverse matrices* and the solution of *simultaneous equations*.

Given A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then the determinant of the matrix A

I A I = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

I A I = ad –bc

Example:

Given A = $\begin{bmatrix} 7 & 2 \\ 9 & 3 \end{bmatrix}$, evaluate the determinant

I A I = $\begin{bmatrix} 7 & 2 \\ 9 & 3 \end{bmatrix}$

I A I = (7 x 3) – (2 x 9)

I A I = 21-18

I A I = 3

Hence, the determinant of the matrix A is 3.

When a matrix has a zero determinant, the matrix is called a *singular matrix*. Any matrix which is not singular is said to be *non-singular*.

Given A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

I A I = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

I A I = ad – bc = 0

Example:

Given A = $\begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix}$

I A I = $\begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix}$

I A I = (8 x 5) – (4 x 10)

I A I = 40 – 40

I A I = 0

2 x 2 Adjoint Matrix

A cofactor matrix *C* of a matrix is the square matrix of the same order as *A* in which the elements of *A* are replaced by its cofactor *C* then multiplying the non-leading diagonal elements by -1. This changes the signs of the non-leading diagonal.

Given A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The cofactor C of A is C = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example:

Given A = $\begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix}$, state the adjoint of A

A*adjoint* = $\begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix}$

C = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

C = $\begin{bmatrix} 5 & -4 \\ -10 & 8 \end{bmatrix}$

Hence, the adjoint matrix of A is $\begin{bmatrix} 5 & -4 \\ -10 & 8 \end{bmatrix}$

Inverse of 2 x 2 Matrix

Given A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$

$= \frac{1}{IAI}$ x A *adjoint*

Then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$AA^{-1} = A^{-1} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ (identity matrix)

If $ad - bc = 0$ then A^{-1} cannot be found and A is a *singular matrix*.

If $ad - bc \neq 0$ then A^{-1} can be found and A is a *non-singular matrix*.

Example:

Determine the inverse of the matrix A = $\begin{bmatrix} 8 & 4 \\ 3 & 5 \end{bmatrix}$

Solution:

The *determinant* of matrix A:

I A I = (8 x 5) – (4 x 3)
I A I = 40 – 12
I A I = 28

A*adjoint* = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ -3 & 8 \end{bmatrix}$

$A^{-1} = \frac{1}{IAI}$ x A *adjoint*
$= \frac{1}{28}$ x $\begin{bmatrix} 5 & -4 \\ -3 & 8 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{28} \times 5 & \frac{1}{28} \times (-4) \\ \frac{1}{28} \times (-3) & \frac{1}{28} \times 8 \end{bmatrix}$

$= \begin{bmatrix} \frac{5}{28} & \frac{-4}{28} \\ \frac{-3}{28} & \frac{8}{28} \end{bmatrix}$

Alternatively, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{(8 \text{ x } 5) - (4 \text{ x } 3)}\begin{bmatrix} 5 & -4 \\ -3 & 8 \end{bmatrix}$

$= \frac{1}{40-12}\begin{bmatrix} 5 & -4 \\ -3 & 8 \end{bmatrix}$

$= \frac{1}{28}\begin{bmatrix} 5 & -4 \\ -3 & 8 \end{bmatrix}$

$= \begin{bmatrix} \frac{5}{28} & \frac{-4}{28} \\ \frac{-3}{28} & \frac{8}{28} \end{bmatrix}$

Solution of Simultaneous Equations

$AX = B$
$A^{-1}AX = A^{-1}B$
$IX = A^{-1}B$
$X = A^{-1}B$

The solution to any simultaneous system is simply $X = A^{-1}B$. A pair of simultaneous equations can be solved by writing the equations in the form of a matrix and then solve the matrix equation using the *matrix method*.

Proving $AA^{-1} = A^{-1}A = I$

Given A $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(i) Then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

So $AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ x $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} ad - bc & -ab + ba \\ cd - dc & -cb + da \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} ad - bc & ab - ab \\ cd - dc & ad - bc \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

$= \begin{bmatrix} \frac{ad-bc}{ad-bc} & \frac{0}{ad-bc} \\ \frac{0}{ad-bc} & \frac{ad-bc}{ad-bc} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore, $AA^{-1} = I$

(ii) Also, $A^{-1}A = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & bd-bd \\ ac-ac & ad-bc \end{bmatrix}$

$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$= \begin{bmatrix} \frac{ad-bc}{ad-bc} & \frac{0}{ad-bc} \\ \frac{0}{ad-bc} & \frac{ad-bc}{ad-bc} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore, $A^{-1}A = I$

Hence, $AA^{-1} = A^{-1}A = I$

The Identity Matrix for Multiplication

Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

And $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then $AI = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a\times 1 + b\times 0 & a\times 0 + b\times 1 \\ c\times 1 + d\times 0 & c\times 0 + d\times 1 \end{bmatrix}$

$= \begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore, $AI = A$

And $IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} 1\times a + 0\times c & 1\times b + 0\times d \\ 0\times a + 1\times c & 0\times b + 1\times d \end{bmatrix}$

$= \begin{bmatrix} a+0 & b+0 \\ 0+c & 0+d \end{bmatrix}$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Therefore, $IA = A$

Hence, $AI = IA = A$

And, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix for the multiplication of 2 x 2 matrices.

Simultaneous Equations

Using a matrix method, solve the simultaneous equations:

3x + 2y = 13
X – 2y = -1

Given 3x + 2y = 13
X – 2y = -1

Re-write in matrix form, $AX = B$, $\begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \end{pmatrix}$

Multiply each side by A^{-1} $A^{-1}AX = A^{-1}B$

So $IX = A^{-1}B$
And $X = A^{-1}B$

The matrix equation $X = A^{-1}B$ allows to solve the pair of simultaneous equations after calculating the inverse of A, A^{-1}.

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{3\times(-2)-2\times 1}\begin{pmatrix} -2 & -2 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{(-6)-2}\begin{pmatrix} -2 & -2 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{-8}\begin{pmatrix} -2 & -2 \\ -1 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{-2}{-8} & \frac{-2}{-8} \\ \frac{-1}{-8} & \frac{3}{-8} \end{pmatrix}$

$= \begin{pmatrix} \frac{2}{8} & \frac{2}{8} \\ \frac{1}{8} & \frac{3}{-8} \end{pmatrix}$

Solving $X = A^{-1}B$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{8} & \frac{2}{8} \\ \frac{1}{8} & \frac{3}{-8} \end{pmatrix}\begin{pmatrix} 13 \\ -1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{8}\times 13 & \frac{2}{8}\times -1 \\ \frac{1}{8}\times 13 & \frac{3}{-8}\times -1 \end{pmatrix} = \begin{pmatrix} \frac{26}{8} & -\frac{2}{8} \\ \frac{13}{8} & +\frac{3}{8} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{26}{8} & -\frac{2}{8} \\ \frac{13}{8} & +\frac{3}{8} \end{pmatrix} = \begin{pmatrix} \frac{24}{8} \\ \frac{16}{8} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Equating corresponding elements, x=3, y=2
Hence, the solution of the simultaneous equations 3x + 2y = 13 and X – 2y = -1is *x=3* and *y=2*.

APPLICATION OF MATRICES IN REAL LIFE CONTEXTS

BUSINESSES

This section is meant to prove that matrices are not only for use by “math nerds”; wittingly or unwittingly, matrices have exhaustive applications in every-day life. This section demonstrates but a few cogent examples. Irrespective of the nature or size of any business venture, for instance, matrices are essential tools. Even from the outset of a business idea, matrices can be used to establish the viability, profitability and sustainability of a business, whether that business be a hospital, an educational institution, a retail or wholesale business, energy exploration and generation enterprise, service providers, or even a budget on a national scale. In businesses one may have multiple inputs and multiple outputs which will offer a degree of predictability. These input/output variables are independent variables which will examine the impact on the dependent variables. For instance, independent variables may include the attitude of staff (loyalty, reliability, honesty, courtesy, efficiency), quality of products, prices, variety of products, and sourcing dependability. Matrices will determine the impact these will have on the dependent variable which is the business itself. In this sense, the independent variables can be represented in the form of a matrix to determine the impact on the profitability and sustainability of the business. With the use of matrix representation, a business can adjust perceived weaknesses in the independent variables. Motivational factors can, for instance, be used to build greater efficiency in workers and management. The smallest of business organizations that has a computer, stores its information and data on spreadsheets. For instance, the operator of a bicycle shop sells three types of bicycles (children, ladies, and gents) in two colours, blue and pink. The operator bought a computer program that calculates the cumulative total of bicycles in each color sold per week and the income earned from each item. The program consists of four types of spreadsheets.

Spreadsheet 1:

This spreadsheet is to enter the number of bicycles sold for each day. The following tables show an example of spreadsheet 1.

Day 1:

	A	B	C	D
1		children	ladies	gents
2	pink	10	16	4
3	blue	11	7	18

Day 2:

	A	B	C	D
1		children	ladies	gents
2	pink	11	12	1
3	blue	13	9	14

Spreadsheet 2:

The spreadsheet below displays the cumulative total of items sold. When an entry is made into any of the daily sales spreadsheet (spreadsheet 1), there is an automatic update in the cumulative total of items represented in spreadsheet 2. The programming of the software involves matrices. The following table shows an example of spreadsheet 2.

	A	B	C	D
1		children	ladies	gents
2	pink	21	28	5
3	blue	24	16	32

The programming involves the following:
Day 1 daily sales are translated into a matrix:

$$\begin{pmatrix} 10 & 16 & 4 \\ 11 & 7 & 18 \end{pmatrix}$$

Day 2 daily sales are also translated into a matrix:

$$\begin{pmatrix} 11 & 12 & 1 \\ 13 & 9 & 14 \end{pmatrix}$$

When each of the different days' totals of each bicycles are added, the cumulative totals of each item are obtained. The operation performed is a *matrix addition*:

$$\begin{pmatrix} 10 & 16 & 4 \\ 11 & 7 & 18 \end{pmatrix} + \begin{pmatrix} 11 & 12 & 1 \\ 13 & 9 & 14 \end{pmatrix}$$
$$= \begin{pmatrix} 21 & 28 & 5 \\ 24 & 16 & 32 \end{pmatrix}$$

Spreadsheet 3:

This spreadsheet is the price list and it is used to enter the cost of the items. The following table shows an example of spreadsheet 3.

	A	B	C
1		pink	Blue
2	Children	\$8000	\$8500
3	Ladies	\$14000	\$13000
	Gents	\$16000	\$18000

Spreadsheet 4:

To keep track of the cumulative income of each item by manual calculations can be tedious and time-consuming. This spreadsheet displays the cumulative income from each item.

	A	B	C	D
1		children	Ladies	gents
2	Pink	\$168,000	\$392,000	\$80,000
3	Blue	\$204,000	\$208,000	\$576,000
	Total	\$372,000	\$600,000	\$656,000

With the use of the computer program, once the number of items is entered into the daily sales (spreadsheet 1), the cumulative total of items automatically updates spreadsheet 2 and the cumulative spreadsheet for each item is displayed on spreadsheet 4. The computer program uses *matrix multiplication* to derive the cumulative income from each item.

For example,
For children' bicycles:

$$(21 \quad 24)\begin{pmatrix} 8000 \\ 8500 \end{pmatrix} = (168000 + 204000) = (372000)$$

That is, the total sales for children's bicycle is \$372,000

CRYPTOGRAPHY

The internet has become the super highway to conduct business, make purchases, and for money transfer. Therefore, confidential information such as passwords, bank account numbers, and credit card information need to be protected. Many websites use cryptography as a form of internet security. Cryptography is a system where communication can remain private and confidential. Cryptography consists of encryption and decryption. Encryption is the coding of data into a form that cannot be read without knowing the code. Decryption is the changing of the encrypted data into a readily readable form. Cryptography uses a *matrix* to encode a message and the coded message received is decoded by the *inverse of the matrix*. Each letter in the alphabet is assigned to a number. A is 1, B is 2, C is 3, etc. and the space between two words is assigned 27 since there are only 26 letters in the English alphabet. For instance, if the word "MATH" is to be sent as an encrypted message, the numbers that are assigned to the word "MATH" is 13, 1, 20, 8.

These numbers are put in a matrix form:

$$\begin{pmatrix} 13 & 20 \\ 1 & 8 \end{pmatrix}$$

The matrix that represents the word "MATH" is coded using another matrix called the coding matrix which can be any matrix that is compatible with the matrix that represents the word "MATH". If the coding matrix is $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$, it is multiplied as follows:

$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \text{x} \begin{pmatrix} 13 & 20 \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 43 & 92 \\ 71 & 148 \end{pmatrix}.$$

The receiver gets the message as a string of numbers 43, 92, 71, 148 along with the coding matrix. The computer's software translates the string of numbers by converting them into a matrix and pre-multiplying this matrix by the inverse of the coding matrix. The numbers obtained from the matrix product is assigned the letters of the alphabet to give the original message which is easily read.

Inverse of coding matrix x $\begin{pmatrix} 43 & 92 \\ 71 & 148 \end{pmatrix}$

$= \begin{pmatrix} 13 & 20 \\ 1 & 8 \end{pmatrix}$

GRAPHICS

Transformation matrices are commonly used in computer graphics and image processing. Matrices are used in computer-generated images that have a reflection and distortion effect such as light passing through rippling water or waves. Hardesty (2013) states: "One of the areas of computer science in which matrix multiplication is particularly useful is graphics, since a digital image is basically a matrix to begin with: the rows and columns of the matrix correspond to the rows and columns of pixels, and the numerical entries correspond to pixels' color values."Matrices may also help to process digital video and digital sound. This is by no means restricted to animated films, photography, and computer graphics. The genius of Leonardo da Vinci's *Mona Lisa* has been confirmed using matric analysis.

GAME THEORY AND ECONOMICS

Fudenberg (1983) states: "In game theory and economics, the pay-off matrix encodes the pay-off for two players, depending on which out of a given (finite) set of alternatives the players choose." In casinos, say, the casino establishes how many players will win and how much they will win, which in turn would establish the overall profit of the casino itself. In other words, the amount gambled by any number of people would determine how much pay-out will be made by ratio of investment to reward percentage-wise. Because of these pre-determined matrix encoding, it is extremely rare to find a casino which loses, thus the saying "the house always wins".

PHYSICS

Matrices can be used to study electrical circuits by calculating battery power outputs. Riley et al. (1992) stipulate: "They [matrices] are needed for describing mechanical vibrations, and oscillations in electrical circuits." The almost ubiquitous cell-phone is built around quantum mechanics and functions on the foundation of matrices. And yet, for all its frequency of use in our every-day lives, we hardly think of matrices when we use cell phones, now an essential part of ordinary life. Weinberg (1995) puts this another way: "Matrix serves as a key tool for describing the scattering experiments that form the cornerstone of experimental particle physics…which encodes all information about the possible interactions between particles."

Conclusion

The examples above are not at all exhaustive. Matrices are also used to determine and graphically represent population and demographic changes/shifts, disease spread, hospitalization and mortality rates, vaccine efficacy, deforestation and re-forestation rates, environmental degradation, and migration rates. These, separately or collectively, determine the quality of life locally, nationally, and internationally. We are already living witnesses to human ignorance or indifference to the destruction of our environment.

REFERENCES

Anton, Howard, 2010. *Elementary Linear Algebra (10th ed.)*, John Wiley & Sons, p. 414 ISBN 978-0-470- 45821-1

Bohm, Arno, 2001. *Quantum Mechanics: Foundations and Applications, Springer* ISBN 0-387-95330-2

Branson, Richard, 1970. *Matrix Methods: An Introduction,* New York: Academic Press, LCCN 70097490

Fudenberg, Drew; Tirole, Jean, 1983. *Game Theory*, MIT Press

Godsil, Chris; Royle, Gordon, 2004. *Algebraic Graph Theory, Graduate Texts in Mathematics*, 207, Berlin, DE; New York, NY: Springer-Verlag ISBN 978-0-387-95220-8

Hardesty, Larry, 2013. *Matrices: Concepts Familiar From Grade-School Algebra Have Broad Ramifications In Computer Science*.

Horn, Roger A.; Johnson, Charles R. 2012. *Matrix Analysis (2nd ed.)*, Cambridge University Press, p. 17 ISBN 978-0-521-83940-2

Pop; Furdui, 2017. *Square Matrices of Order 2.* Springer International Publishing. ISBN 978-3-319- 54938-5

Riley, Kenneth F.; Hobson, Michael P.; Bence, Stephen J. 1997. *Mathematical Methods for Physics and Engineering*, Cambridge University Press ISBN 0-521-55506-X

Stinson, Douglas R. 2005. *Cryptograph, Discrete Mathematics and its Applications*, Chapman & Hall/CRC, ISBN 978-1-58488-508-5

Weinberg, Steven, 1995. *The Quantum Theory of Fields. Volume 1: Foundations*, Cambridge University Press ISBN 0-521-55001-7.

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