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Research Article

DESIGN OF CIRCULAR PIEZOELECTRIC ACTUATOR FOR THE MICRO ADAPTIVE MIRRORS

***Dong-Chan Lee**

IAE, 175-28, Goan-ro 51 beon-gil, Baegam-myeon, Cheoin-gu, Yongin-si, Gyeonggi-do, 17180, Korea

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Abstract

This paper presents the numerical modeling of a piezoelectric actuator. The vibro-elastic piezoelectric actuator has the layered structures. Applying the equivalent parameters based on the plate/shell theories to the multilayered structures, it is possible to investigate the geometric and material characteristics of multilayered systems and estimate their structural performances. And the optimization designs for the geometric dimensions or material properties of multilayered systems on the layered model can easily be achieved.

Keywords: Multilayered systems, Flexural stiffness, Structural damping, Bimorph model.

INTRODUCTION

In recent years, there have been many increases in the development of piezoelectric actuating and control systems in many micro-engineering fields. One reason for these applications is that it is possible to create the systems with the suitable control in the macro- or micro-scale ranges. These systems are capable of adapting themselves to varying operating conditions. The sensing and actuating mechanisms are possible from the direct and inverse piezoelectric effects; when a mechanical load is applied to a piezoelectric material, an electrical voltage is generated and conversely, when an electrical field is applied, a mechanical deformation is induced. The multilayered systems are commonly used in macro-scale or micro-scale applications such as active structural damping or excitation and precision positioning systems. The formulations of actuator configurations which are composed of the piezoelectric layers and non-piezoelectric layers are very important for the conceptual design. The geometric configuration of the layered system is very important for the structural design with low cost, high performance and quality. Particularly, some layered systems will be designed to minimize weight and to achieve a high stiffness such as sandwich beams or plates. For the layered systems subjected to bending, the effective mechanical properties such as flexural stiffness can be determined using the rule of mixtures and are dependent on the layer configurations. These devices are sufficient for determining the quasi-static behavior of the system since the layers are relatively thin. For the development of multilayered systems, the main assumptions of plate/shell theories, which allow for transverse shear deformation effects, are that the displacements are small compared to the plate thickness, the stress normal to the plate mid-surface is negligible, and normal to the mid-surface remain straight but not necessarily normal to the mid-surface after deformation. Ross-Ungar-Kerwin equations (Wang and Cross, 1999), single element modeling based on variation asymptotical or symmetric theory (Mo *et al.,* 2006; Hwang and Park, 1993) and transfer matrix procedure (Prasad *et al.,* 2006; De Abreu *et al.,* 2004; Ugural and Fenster, 1995; Agnes, 1995) have been developed to describe the layered material treatment.

Figure 1 Micro adaptive mirror

Corresponding Author:** ***Dong-Chan Lee
IAE, 175-28, Goan-ro 51 beon-gil, Baegam-myeon, Cheoin-gu, Yongin-si, Gyeonggi-do, 17180, Korea.

This paper presents this effective approach to estimate the layer configurations and flexural stiffness of multilayered materials. The concept can be developed to describe the flexural stiffness of more complex multilayer configurations containing these materials, with different layer thickness and spaces and with symmetric or asymmetric layer distributions. It may be possible to estimate the physical and mechanical properties and multi-material multilayered structures.

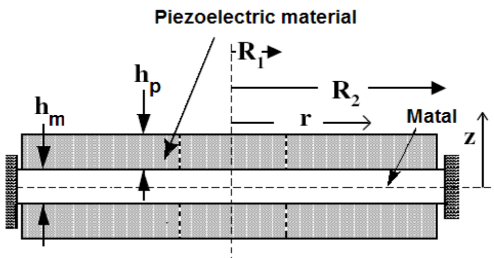
Theoretical considerations for the circular piezoelectric actuator

In the formulations of piezoelectric actuator model, the following assumptions are made (Akhras and Li, 2007; Mo *et al.,* 2006; Prasad *et al.,* 2006; De Abreu *et al.,* 2004; Crandall *et al.,* 1978; Timoshenko and Gere, 1963); the piezoelectric layers is initially flat, its thickness is very small compared to lateral dimensions, and the piezoelectric layers are perfectly bonded to other materials. The formulation is also restricted to linear elastic material behavior in small deformations and strains, and the formulation uses the Kirchhoff assumption in which the transverse normal remains straight after deformation, when we neglect tensile stress perpendicular to the mid-surface (σ_{zz}), the transverse loading is assumed to be applied at the mid-surface and rotate so that they always remain perpendicular to the mid-surface. The equilibrium equations of the axi-symmetric plate shown in figure 5-(a) can be given by.

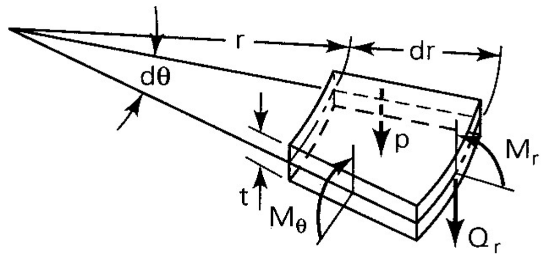
$$\frac{dN_r}{dr} + \frac{N_r - N_\theta}{r} = 0 \tag{13}$$

$$\frac{dM_r}{dr} + \frac{M_r - M_\theta}{r} = Q_r \tag{14}$$

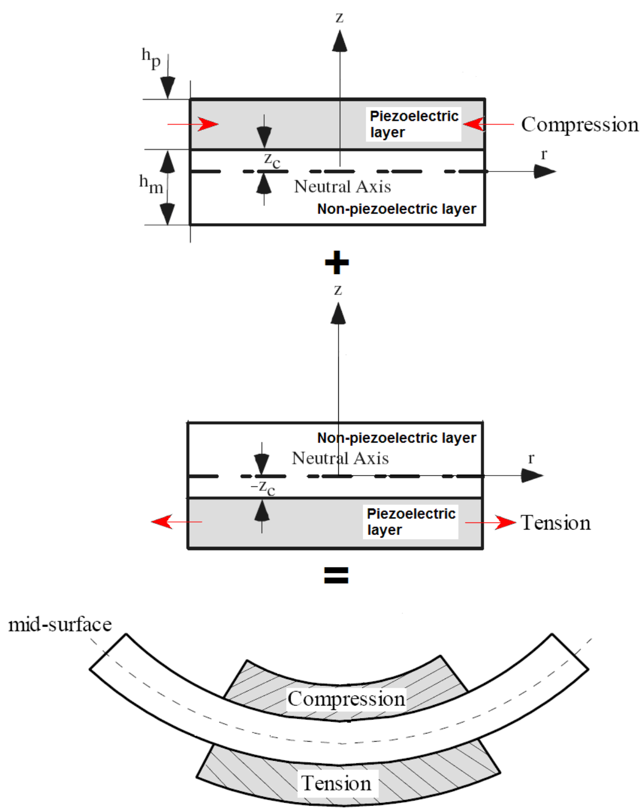
$$\frac{dQ_r}{dr} + p + \frac{Q_r}{r} = \frac{1}{r}\frac{d}{dr}(r\,Q_r) + p = 0 \tag{15}$$



(a) A schematic cross-section of axi-symmetric piezoelectric actuator



(b) Axi-symmetrically loaded circular plate element



(c) Expansion of piezoelectric layers

Figure 1. Schematic piezoelectric circular plates

where N_r and N_θ are the force resultants in the radial and circumferential directions, M_r and M_θ are the moment resultants, and Q_r is the transverse shear force resultant.

Substituting eq. (14) into eq. (15), the equilibrium equations can be reduced to eq. (16).

$$\frac{1}{r}\frac{d}{dr}\left(r\,\frac{dM_r}{dr}+M_r-M_\theta\right)+p=0 \tag{16}$$

The radial and conferential strain-displacement relationships from Kirchhoff's plate theory are

$$\begin{aligned}\varepsilon_r &= \varepsilon^0{}_r + z\,\rho_r\\ \varepsilon_\theta &= \varepsilon^0{}_\theta + z\,\rho_\theta\end{aligned} \tag{17}$$

where $\rho_r=-\left(d^2w/dr^2\right)$ and $\rho_\theta=-\left(1/r\right)\left(dw/dr\right)$ are the radial and circumferential curvatures. For the unimorph plate, z is replaced by $z-z_c$. The strains in the reference plane $(z=0)$ are given by.

$$\begin{aligned}\varepsilon^0{}_r &= du_0/dr = 0\\ \varepsilon^0{}_\theta &= u_0/dr = 0\end{aligned} \tag{18}$$

The constitutive equations for the piezoelectric material are given by.

$$\begin{aligned}\varepsilon_r &= C^E{}_{11}\left(\sigma_r-\upsilon\,\sigma_\theta\right)-d_{31}\,E_3\\ \varepsilon_\theta &= C^E{}_{11}\left(\sigma_\theta-\upsilon\,\sigma_r\right)-d_{31}\,E_3\\ D_3 &= -d_{31}\left(\sigma_r+\sigma_\theta\right)+e_{33}{}^T E_3\end{aligned} \tag{19}$$

where $\upsilon\left(=-C^E{}_{12}/C^E{}_{11}\right)$ is Poisson's ratio, σ is the stress, $e_{33}{}^T$ is the permittivity of the piezoelectric, d_{31} is the piezoelectric constant, D_3 is the charge density, $E_3\left(=V_{ac}/h_p\right)$ is the strength of the electric field and $C^E{}_{11}$ is the elastic compliance of the piezoelectric layer.

The radial and circumferential stress can be expressed for the piezoelectric layers and non-piezoelectric layers as follows.
For the piezoelectric layer;

$$\begin{aligned}\sigma_{r\,p} &= \frac{1}{C^E{}_{11}\left(1-\upsilon^2\right)}\left(\varepsilon_r+r\,\varepsilon_\theta+\left(1+\nu\right)d_{31}\,E_3\right)\\ \sigma_{\theta\,p} &= \frac{1}{C^E{}_{11}\left(1-\upsilon^2\right)}\left(\nu\,\varepsilon_r+\varepsilon_\theta+\left(1+\nu\right)d_{31}\,E_3\right)\end{aligned} \tag{20}$$

For the non-piezoelectric layer,

$$\begin{aligned}\sigma_{r\,m} &= \frac{1}{C_m\left(1-\upsilon^2\right)}\left(\varepsilon_r+r\,\varepsilon_\theta\right)\\ \sigma_{\theta\,m} &= \frac{1}{C_m\left(1-\upsilon^2\right)}\left(\nu\,\varepsilon_r+\varepsilon_\theta\right)\end{aligned} \tag{21}$$

where $C_m\left(=1/E_m\right)$ is the elastic compliance of the non-piezoelectric layer and υ is Poisson's ratio of the non-piezoelectric layer.

If there are two pairs of piezoelectric elements for both sides of non-piezoelectric element, the deflection due to the bending moments and shear forces will be doubled because the conditions are symmetric. Therefore, on the base of the unimorph piezoelectric actuating mechanism, the radial and circumferential force resultants, N_r and N_θ, and the moments per unit length M_r and M_θ in the bimorph piezoelectric actuating mechanism are given by.

$$N_r = \int_{z_c}^{h_p+z_c}\left(\sigma_{r\,p}\Big|_{tension} - \sigma_{r\,p}\Big|_{compression}\right)dz + \int_{-(h_m-z_c)}^{z_c}\left(\sigma_{r\,m}\Big|_{tension} - \sigma_{r\,m}\Big|_{compression}\right)dz \qquad (22)$$

$$N_\theta = \int_{z_c}^{h_p+z_c}\left(\sigma_{\vartheta\,p}\Big|_{tension} - \sigma_{\vartheta\,p}\Big|_{tcompression}\right)dz + \int_{-(h_m-z_c)}^{z_c}\left(\sigma_{\theta\,m}\Big|_{tension} - \sigma_{\theta\,m}\Big|_{compression}\right)dz$$

$$M_r = 2\int_{z_c}^{h_p+z_c}\sigma_{r\,p}\left(z - z_c\right)dz + 2\int_{-(h_m-z_c)}^{z_c}\sigma_{r\,m}\left(z - z_c\right)dz \qquad (23)$$

$$M_\theta = 2\int_{z_c}^{h_p+z_c}\sigma_{\theta\,p}\left(z - z_c\right)dz + 2\int_{-(h_m-z_c)}^{z_c}\sigma_{\theta\,m}\left(z - z_c\right)dz$$

In the absence of external force in eq. (15), the equilibrium equations for any radial section of the axi-symmetric plate may be given by zero and for the transverse direction,

$$\frac{A}{6\,C_m\left(S^E{}_{11}\,h_m + h_p\,E\right)C^E{}_{11}\left(1-\upsilon^2\right)}\left\{\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right\} = 0 \qquad (24)$$

where $A = 4{h_m}^3 h_p C^E{}_{11}E + 6{h_m}^2{h_p}^2 C^E{}_{11}E + 4h_m{h_p}^3 C^E{}_{11}E + {h_p}^4{C_m}^2 + {h_m}^4\left(C^E{}_{11}\right)^2$

As a result, the piezoelectric equations may explicitly not appear explicitly in the governing displacement equations, but may arise from the matching conditions at the interface $\left(r = R_1\right)$ and rim $\left(r = R_2\right)$. The general equations of the radial and transverse directions are given by.

$$\frac{dw}{dr} = \theta(r) = b_1\,r + \frac{b_2}{r} \qquad (25)$$

For the inner annular plate without the piezoelectric layer, the moment per unit length, ${M_r}^{(2)}$ and ${M_\theta}^{(2)}$ due to the bimorph piezoelectric are

$${M_r}^{(2)} = 2\int_{-(h_m-z_C)}^{z_C}\left(z - z_C\right)\sigma_{r\,m}\,dz \qquad (26)$$

$${M_\theta}^{(2)} = 2\int_{-(h_m-z_C)}^{z_C}\left(z - z_C\right)\sigma_{\theta\,m}\,dz$$

The boundary conditions include the finite values at the origin, the clamped conditions at the support and a amatching condition at the interface $\left(r = R_1\right)$.

$$\theta\left(0\right) < \infty\,,\ \theta\left(R_2\right) = 0\,,\ \theta^{(1)}\left(R_1\right) = \theta^{(2)}\left(R_1\right)$$

$$w\left(R_2\right) = 0\,,\ w^{(1)}\left(R_1\right) = w^{(2)}\left(R_1\right) \qquad (27)$$

$${M_r}^{(1)}\left(R_1\right) = {M_r}^{(2)}\left(R_1\right)$$

The general solution of transverse displacement can be given by.

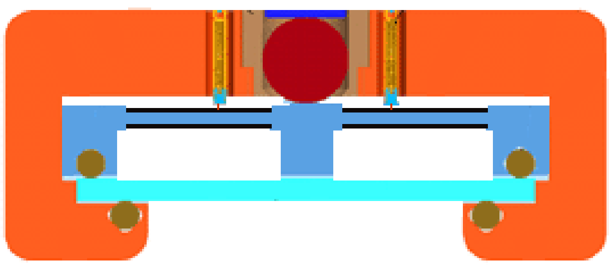
$$w_{pmm}(r)=\begin{cases}\dfrac{c_1\left\{2R_1^2\ln\left(R_1/R_2\right)+\left[1-\left(R_1/R_2\right)^2\right]r^2\right\}V}{c_2-c_3\left(R_1/R_2\right)^2+\frac{1}{2}h_p^4C_m^2(1+\upsilon)\left(R_1/R_2\right)^4} & r\le R_1\\[2ex] \dfrac{c_1\left\{2R_1^2\ln(r)-R_1^2[2\ln(R_2)-1]-\left(R_1/R_2\right)^2r^2\right\}V}{c_2-c_3\left(R_1/R_2\right)^2+\frac{1}{2}h_p^4C_m^2(1+\upsilon)\left(R_1/R_2\right)^4} & R_1<r\le R_2\end{cases} \qquad (28)$$

where $c_1=6d_{31}\,h_m\,C^E{}_{11}\,C_m\,(h_m+h_p)$

$$c_2=4h_m^3h_pC^E{}_{11}C_m+6h_m^2h_p^2C^E{}_{11}C_m+4h_mh_p^3C^E{}_{11}C_m+\tfrac{1}{2}h_p^4C_m^2(1+\upsilon)+2h_m^4\left(C^E{}_{11}\right)^2/(1+\upsilon)$$

$$c_3=4h_m^3h_pC^E{}_{11}C_m+6h_m^2h_p^2C^E{}_{11}C_m+4h_mh_p^3C^E{}_{11}C_m+h_p^4C_m^2(1+\upsilon)$$

Because the motions of a piezoelectric actuated mirror are driven by piezoelectric effects, a piezoelectric actuated mirror is called to a piezoelectric mirror. A geometric model of a piezoelectric mirror is shown in figure 6. A piezoelectric mirror can be represented as a combination set of five parts as shown in figure 6; (1) piezoelectric metal membrane, (2) mirror membrane for lateral balance, (3) mirror central post, (4) reflective mirror for ray reflection. A piezoelectric metal membrane is a round-shaped basic plate with two piezoelectric elements soldered symmetrically to its faces. The mirror central post can have a varying or a constant cross-section. A mirror membrane is a round-shaped plate, and the rim is assumed to be rigid. Four parts are clamped on the rim as a stationary frame of reference. When the electrical voltages are applied to the piezoelectric elements, all parts of the assembly are deformed in the equilibrium state.



(a) Section

(b) Component configuration

Figure 2. Schematic configuration of circular piezoelectric actuator

The transverse deflection is due to the piezoelectric radial deformation in the radial direction. From the assumption that the transverse deflections due to the mechanical uniformly pressure on the non-piezoelectric plate and the piezoelectric element on the non-piezoelectric plate are equal, the pressure on the non-piezoelectric element can be given from eq. (28) and the transverse deflection equation of axi-symmetrically loaded circular plates (Akhras and Li, 2007; Mo *et al.,* 2006; Prasad *et al.,* 2006).Under the piezoelectric surface stretching and contraction of the circular plate of the metal membrane, when all parts of the assembly are deformed in the equilibrium state, the metal membrane bends and through the rod, pushes or pulls the central part of the reflective mirror membrane on the support plate. The global transverse deflection at the reflective mirror membrane is given by,

$$w_{res}=w_{pmm}-w_{mss}-w_{mcp}-w_{rmm} \qquad (29)$$

where w_{pmm} is the transverse deflection of piezoelectric metal membrane, w_{mss} is the axial deformation of mirror screw support, w_{mcp} is the axial deformation of mirror central post and w_{rmm} is the reflective mirror membrane.

In eq. (29), the elongation of mirror screw support ΔH_{mss} and mirror central post ΔH_{mcp} due to the piezoelectric metal membrane can be given by.

$$w_{mss} = \Delta H_{mss} = \int_0^{h_{mss}} \frac{P}{A_{mss}(z) E_{mss}} dz = \frac{P}{E_{mss}} \int_0^{h_{mss}} \frac{dz}{A_{mss}(z)} \qquad (30)$$

$$w_{mcp} = \Delta H_{mcp} = \int_0^{h_{mcp}} \frac{P}{A_{mcp}(z) E_{mcp}} dz = \frac{P}{E_{mcp}} \int_0^{h_{cp}} \frac{dz}{A_{mcp}(z)} \qquad (31)$$

where E_{mss} and E_{mcp} are the elastic modulus of mirror screw support and mirror central post, and A_{mss} and A_{mcp} are the areas of cross sections of them.

In the reflective mirror membrane with the concentrated force acting at the center of the plate and the outer edge which is constrained against rotation, the transverse deflection is given by.

$$w_{rmm}(r) = \frac{P\left[\left(R_{rmm\_o}{}^2 - r^2\right) + 2r^2 \ln\left(r \big/ R_{rmm\_o} \right)\right]}{16\,\pi\, D_{rmm}} \qquad (32)$$

where the flexural rigidity of reflective mirror membrane is $D_{rmm} = E_{rmm} h_{rmm}{}^3 \big/ 12\left(1 - v_{rmm}{}^2\right)$.

The concentrated force P transferred to mirror screw support, mirror central post in figure 6 can be given by,

$$P = \frac{32\,\pi\, D_{pmm}}{\left[\left(R_2{}^2 - R_1{}^2\right) + 2R_1{}^2 \ln\left(R_1 \big/ R_2 \right)\right]} \frac{c_1\, R_1{}^2 \ln\left(R_1 \big/ R_2 \right) V}{c_2 - c_3\left(R_1 \big/ R_2 \right)^2 + 1\big/2\, h_p{}^4 C_m{}^2 (1+\upsilon)\left(R_1 \big/ R_2 \right)^4} \qquad (33)$$

Conclusion

This paper presents the general modeling method of multilayer structure with *m*-layers. A model describing the deflection of a piezoelectric multi-morph structure can be derived by using the basic mechanics principles of static equilibrium and strain compatibility between the interfacial layers. Using this formulation, the layer selection and distribution is easy in designing the layered model. The change from one configuration to another can be represented by a change of the related modification of the assignments of equivalent material parameters to the different sub-layers. And, the geometric dimensions such as thickness, width and height in the materials can be studied for the improvement of static or dynamic stiffness. In the numerical equations, the nominal material properties of piezoelectric elements and seam elements are generally used, but they have the probabilistic distributions of material properties. In the consideration of the probabilistic distributions of material properties, the presented equations are available for the structural characteristics..

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