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Research Article

THE ALTERNATIVE PARAMETER SELECTION METHOD AND STEADY-STATE SOLUTIONS IN CORROSION ANALYSIS OF Al-Sn ALLOY INTERACTING SYSTEM

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Abstract

In this work, we have used the infinity-norm penalty function to select the best-fit model parameters which define a system of first order ordinary differential equations that describe the interaction between two corrosion penetration data over a time interval. For this particular problem, the estimated best-fit model parameters are the intrinsic growth rate of the first corrosion population which has the value of -0.5092 followed by the intraspecific coefficient which has the value of -0.1570. The other model parameters are estimated under some simplifying assumptions. The precise steady-state solutions are (0,0), (0, 0.3500), (3.2433, 0) and (1.2817, 2.5664).

Keywords: Parameters, Penalty Function, Corrosion Penetration Data, Steady-State Solution, Best-fit Model Parameters, Intraspecific Coefficient, Intrinsic Growth Rate.

INTRODUCTION

The statistical analysis of binary Al-Sn alloy systems using model equations has attracted the interest of many physicists (Ekuma *et al.,* 2007). Their major conclusion was that, the modeled corrosion penetration rate values generated using the developed model equations were in agreement with the experimental values. Which numerical method can we use to select the best-fit model parameters that define the interacting system of corrosion penetration data? In the vast literatures of corrosion data analysis (Ekuma *et al.,* 2007; Ekuma *et al.,* 2011; Ekuma and Idenyi, 2007; Ekuma *et al.,* 2007; Idenyi and Neife, 2005; Idenyi *et al.,* 2004;Idenyi *et al.,* 2006; Onuchukwu, 2004; Troost *et al.,* 2007; Ulick, 1976; Zhang and Lyon, 1992; Zoccola *et al.,* 1978), it is rare to find previous research outputs which have used a numerical selection method to select best-fit parameters.

METHOD OF SOLUTION

In this study, we will attempt to apply the infinity-norm penalty function to select the best-fit parameters on the simplifying assumption of finding the local minimum from a monotonic sequence of infinity-norm values. This powerful and challenging selection method of model parameters can provide further insights in the capacity building of forecasting analysis, sensitivity analysis, stabilization of the mathematical model of two interacting corrosion penetration data, fractal analysis due to environmental perturbation and stochastic analysis of the same interacting systems to mention a few. In sections 2 and 4, our results will be presented and interpreted. In section 3, a typical corrosion interaction model of Lotka-Volterra type is formulated. In section 5, our core results are discussed while in section 6, our key observations of this numerical study and further extensions are pointed out.

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Results 1: Selection of model parameters

Following Ekaka-a (2009), we have selected the intrinsic growth rate a and used the infinity-norm penalty function to select the intraspecific coefficient b of the first corrosion penetration rate population. In this section, as inthe previous analysis, we shall find those logistic model parameters which minimize the infinity-norm sequence of values. Our calculations are presented below. What do we want to find out? We are interested to find a list of best-fit model parameters of our logistic model that will minimize the agreement between the provided model and our simulated model (Ekuma *et al.,* 2007). Our calculations are presented next.

Table 1. The calculation of the local minimum using the infinity-norm penalty function

Parameter	Calculation of the infinity-norm local minimum			
N	b	ss	a	Infinity-norm
1	-0.1696	3.0024	-0.5092	0.5714
2	-0.1570	3.2433	-0.5092	0.4062
3	-0.1462	3.4829	-0.5092	0.4094
4	-0.1368	3.7222	-0.5092	0.4123
5	-0.1285	3.9626	-0.5092	0.4171
6	-0.1211	4.2048	-0.5092	0.4219
7	-0.1146	4.4433	-0.5092	0.4281
8	-0.1087	4.6845	-0.5092	0.4330

From Table 1, we observe that our expected local minimum 0.4062 occurs when the value of the intraspecific coefficient b is -0.1570. Our next task is to grid around this value of b = -0.1570 until we can find a smaller local minimum if possible (see Table 2 below). From these series of infinity-norm penalty function calculations, it is impossible to find a smaller local minimum than the first local minimum value of 0.4062. To investigate this challenging problem, we would proceed to check if we can find a smaller local minimum within a different parameter space of intra-specific coefficient and steady-state values. From this scenario, it is impossible to find a smaller local minimum than the first local minimum value of 0.4062.

Table 2. The calculation of a further gridding around the local minimum value of 0.4062

Parameter	Calculation of the infinity-norm penalty function			
N	*b*	ss	*a*	Infinity-norm
1	-0.1460	3.4877	-0.5092	0.5699
2	-0.1461	3.4853	-0.5092	0.5685
3	-0.1463	3.4805	-0.5092	0.5670
4	-0.1464	3.4781	-0.5092	0.5656
5	-0.1465	3.4758	-0.5092	0.5641
6	-0.1466	3.4734	-0.5092	0.5627
7	-0.1467	3.4710	-0.5092	0.5613
8	-0.1468	3.4687	-0.5092	0.5598
9	-0.1469	3.4663	-0.5092	0.5584
10	-0.1470	3.4639	-0.5092	0.5570

Next, we consider other parameter spaces of parameter b and steady-state values. By a similar calculation, we obtain Table 3.

Table 3. The calculation of a further gridding around the local minimum value of 0.4062

Parameter	Calculation of the infinity-norm penalty function			
N	*b*	ss	*a*	Infinity-norm
1	-0.1685	3.0220	-0.5092	0.5556
2	-0.1684	3.0238	-0.5092	0.5541
3	-0.1683	3.0255	-0.5092	0.5527
4	-0.1682	3.0273	-0.5092	0.5513
5	-0.1681	3.0291	-0.5092	0.5499
6	-0.1680	3.0310	-0.5092	0.5485
7	-0.1679	3.0328	-0.5092	0.5470
8	-0.1678	3.0346	-0.5092	0.5456
9	-0.1677	3.0364	-0.5092	0.5442
10	-0.1676	3.0382	-0.5092	0.5428
11	-0.1675	3.0400	-0.5092	0.5414

So far, it has not been possible to find a smaller local minimum than 0.4062. Therefore, we would proceed to check if we can find a smaller local minimum with a different parameter space of intraspecific coefficient and steady-state values. Our final infinity-norm penalty function calculations in a bid to find a smaller local minimum are presented below (see Table 4).

Even in this set of intraspecific coefficient and steady-state values, we report that it is impossible to find another smaller local minimum than the first local minimum value of 0.4062. Having systematically embarked upon searching for the local minimum using the infinity-norm penalty function selection method, we report that a further local minimum cannot be found. Therefore, the best model parameters from these several logistic candidate models are *a* = -0.5092 and *b* = -0.1570.

Table 3. A different calculation of a further gridding around the local minimum value 0.4062

Parameter	Calculation of the infinity-norm penalty function			
N	*B*	ss	*a*	Infinity-norm
1	-0.1685	3.2412	-0.5092	0.4066
2	-0.1684	3.2392	-0.5092	0.4078
3	-0.1683	3.2371	-0.5092	0.4090
4	-0.1682	3.2351	-0.5092	0.4102
5	-0.1681	3.2330	-0.5092	0.4114
6	-0.1680	3.2310	-0.5092	0.4126
7	-0.1679	3.2454	-0.5092	0.4063
8	-0.1678	3.2474	-0.5092	0.4063
9	-0.1677	3.2495	-0.5092	0.4063
10	-0.1676	3.2516	-0.5092	0.4064
11	-0.1675	3.2537	-0.5092	0.4064
12	-0.1675	3.2558	-0.5092	0.4064
13	-0.1675	3.2578	-0.5092	0.4064
14	-0.1675	3.2599	-0.5092	0.4065

MATHEMATICAL FORMULATION

Based on the infinity-norm penalty function (Ekaka-a, 2009; Ekuma *et al.,* 2011), a typical corrosion interaction model will take the following form.

$$\frac{dN_1}{dt} = N_1(a - bN_1 - cN_2) \quad (1)$$

$$\frac{dN_2}{dt} = N_2(d - eN_1 - fN_2) \quad (2)$$

where the initial conditions are $N_1(0) = N_{10} > 0$ and $N_2(0) = N_{20} > 0$. Here, the calculated value a is -0.5092 while the infinity-norm penalty function selected value of b is -0.570. The parameter c has an assumed value of -0.12. The intrinsic growth rate has a value of -0.4875 for the second population which is assumed because the first population interacts similarly with the second population. The parameter e having a value of -0.08 is assumed. On the basis of the same assumption, the value of f is -0.15.

Results 2: characterization of steady state solutions

By equating $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ to zero and solving these non-linear equations analytically for the appropriate values of N_1 and N_2, when $a = -0.5092$, $b = -0.1570$, $c = -0.12$, $d = -0.4875$, $e = -0.08$, $f = -0.15$, we will obtain the following steady-state solutions such as $(0, 0)$, $(0, 3.2500)$, $(3.2433, 0)$ and $(1.2817, 2.5664)$.

DISCUSSION OF RESULTS

In this study, we have calculated four steady-state solutions which have the following implications. For the trivial steady-state solution, the two populations of corrosion penetration systems will be driven into the environmental risk of extinction. For the first border steady-state solution (0, 3.2500), the first corrosion penetration population will be driven into extinction while the second corrosion penetration population will survive at its carrying capacity or maximum population value of 3.2500. For the second border steady-state solution (3.2433, 0), the first corrosion penetration rate population will survive at its carrying capacity or value of 3.2433 while the second corrosion penetration population will be driven into extinction. The only positive and unique steady-state solution (1.2817, 2.5664) specifies that a population size value of 1.2817 of the first population and a population size value of 2.5664 of the second population are required for these two populations to co-exist in the context of competition interaction.

CONCLUDING REMARKS AND FURTHER RESEARCH

In this study, we have found four appropriate steady-state solutions which have consistent qualitative characteristics to the four steady-state solutions which we calculated in our previous work and consistent with other reported studies (Ekaka-a, 2009; Ford and Norton, 2009). A further characterization of the stability of the steady-state solutions which we have found in this study is proposed. The steady-state solutions can support further insights into the bifurcation analysis (Ford and Norton, 2009), stabilization of this population system, parameter ranking or sensitivity analysis and stochastic analysis which we did not tackle in this present analysis.

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