International Journal of Science Academic Research
Vol. 04, Issue 11, pp.6570-6574, November, 2023
Available online at http://www.scienceijsar.com

ISSN: 2582-6425

Research Article

STATE SPACE EQUATIONS AND SIMULATION OF MEMS ELECTROSTATIC ACTUATOR TO EQUIVALENT CIRCUIT

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Received 26th September 2023; **Accepted** 20th October 2023; **Published online** 28th November 2023

Abstract

The rapid advancement of Micro-Electro-Mechanical Systems (MEMS) technology has led to the development of various types of actuators with diverse applications. This paper presents a comprehensive study on the state space equations and simulation of MEMS electrostatic actuators using an equivalent circuit model. The key objectives include introducing MEMS electrostatic actuators, developing a simplified equivalent circuit model, deriving mathematical modeling of the equivalent circuit, and conducting simulations of LR and LRC circuits as well as the equation of motion in MATLAB. We followed applied mathematical method using Matlab. The results demonstrate the practicality and accuracy of the proposed modeling and simulation techniques. The insights gained from this study hold significant potential for enhancing the design and performance optimization of MEMS electrostatic actuators in various engineering applications.

Keywords: Micro-Electro-Mechanical Systems,State Space Equations, Equivalent Circuit Model, Simulation.

INTRODUCTION

The capacity of Micro-Electro-Mechanical Systems (MEMS) to combine microscale electrical and mechanical components onto a single chip has completely changed the engineering and technological fields. MEMS actuators, which allow for precise motion and positioning control, are an essential part of these systems. MEMS electrostatic actuators in particular have drawn a lot of interest because of their effectiveness, scalability, and adaptability [1, 2]. The purpose of this research is to derive state space equations for MEMS electrostatic actuators and to construct a simplified equivalent circuit model in order to facilitate a better understanding and practical implementation of these devices. The main goal of this project is to develop a more user-friendly and effective modeling and simulation method for MEMS electrostatic actuators, which will greatly accelerate the design and optimization processes [3,4]. The study is organized as follows: we start off by providing an overview of MEMS electrostatic actuators and the different technical applications that they are important for. We then go on to the mathematical modeling of this circuit, the derivation of state space equations, and the creation of a simplified equivalent circuit model. The outcomes of LR and LRC circuit simulations as well as MATLAB equation of motion simulations are then shown. These findings provide important new information about the behavior and functionality of MEMS electrostatic actuators. Lastly, we wrap off by highlighting the major discoveries and their consequences for the area [5, 6].

MEMS Electrostatic Actuators

The essential components that enable MEMS to carry out physical movements are MEMS electrostatic actuators, sometimes referred to as micro-actuators [7]. Their compact size, affordability, and low power consumption are their advantages. The usefulness of electrostatic actuators is demonstrated by an astounding range of applications. Micro-mirrors, optical gratings, variable capacitors, and micro-accelerometers are a few instances of the uses [8]. A micro mirror device's usage of a parallel-plate electrostatic actuator is simplified in Figure 1 [9].

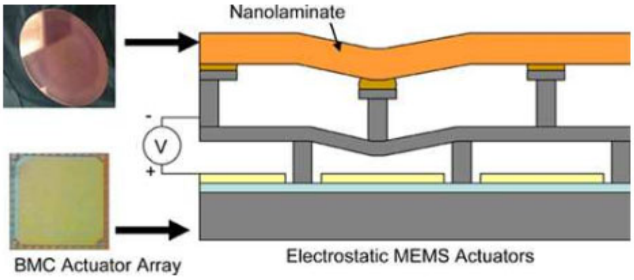

Figure 1. Illustration of an Electrostatic Actuator Used in Micro-mirror Device [9]

Figure 2 illustrates how a simplified model of the electrostatic actuator that includes the crucial system dynamics may be created using this design.

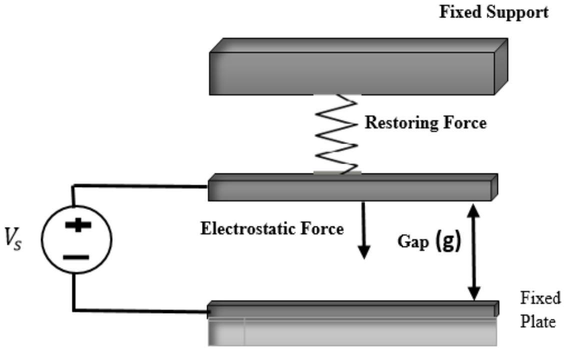

Figure 2. A Simplified Model of the Electrostatic Actuator [4]

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Two moveable plates and one fixed plate are shown as a parallel-plate micro-actuator in Figure 2 in an electric field. The capacitance created between the two plates changes when the movable plate moves off of its initial location. As a result, by adjusting the capacitor's gap voltage, one may alter the movable plate's displacement. However, a pull-in (or snap-down) event will cause the system to become unstable when the gap between the two plates gets closer to two thirds of its initial size. This will drag the movable plate to the fixed plate, instantly reducing the distance to zero [10].

Developing a Simplified Equivalent Circuit Model

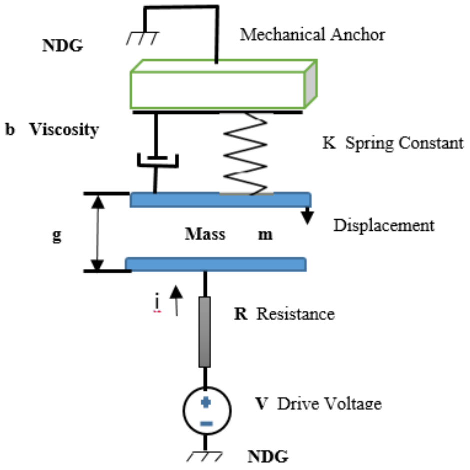

Figure 3. Electrostatic actuator model [1]

A schematic of a parallel-plate electrostatic actuator is shown in Figure 3. The motion of the plate is described by equations (1) to (5) [11, 12].

Electrostatics

$$F = \frac{1}{2}\varepsilon\frac{A}{(g)^2}V_{in} \tag{1}$$

$$Q = \varepsilon\frac{A}{g}V_{in} \tag{2}$$

Equation of motion

$$\ddot{g} = \frac{1}{m}\left(F - b\dot{g} - k(g - g_0)\right) \tag{3}$$

Kirchoff's Laws

$$V = Ri + V_{in} \tag{4}$$

$$i = \frac{d}{dt}(CV_{in}) \tag{5}$$

MEMS Electrostatic Actuator to Equivalent Circuit

By combining (2), (4) and (5) we can write (6) as:

$$\dot{Q} = I = \frac{1}{R}\left[V - \frac{Qg}{\varepsilon A}\right] \tag{6}$$

And by combining (1) and (3) we can write (7) as:

$$\frac{Q^2}{2\varepsilon A} + b\dot{g} + m\ddot{g} + k(g - g_0) = 0 \tag{7}$$

With the initial condition $g = g_0$, we may solve equation (6) to obtain the value of charge, Q, and we can solve equation (7) to obtain the value of gap, g. As coupled differential equations, these two equations can be represented as RL and RLC circuits, respectively. Figure 4 displays the entire equivalent circuit model.

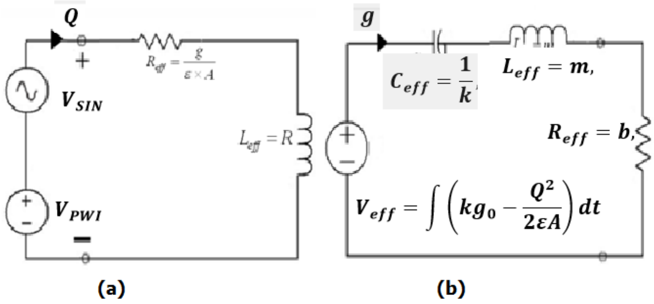
Q
+
$R_{eff} = \frac{g}{\varepsilon \times A}$
V_{SIN}
$L_{eff} = R$
V_{PWI}
(a)
g
$C_{eff} = \frac{1}{k}$
$L_{eff} = m,$
$R_{eff} = b,$
$V_{eff} = \int\left(kg_0 - \frac{Q^2}{2\varepsilon A}\right)dt$
(b)

Figure 4. Equivalent circuit model of electrostatic actuator

Deriving Mathematical Modeling of Equivalent circuit

Kirchhoff's Voltage Law (KVL) provides the following to derive mathematical modeling for an RL circuit in figure (2):

$$\dot{I} = \frac{V - IR}{L} \tag{8}$$

By contrasting equation (6) with equation (8), it can be asserted that the present (I) in the RL circuit is essentially equivalent in numerical terms to the charge (Q) described in equation (6), provided that the effective inductance and effective resistance of the RL circuit are specified as:

$$R_{eff} = \frac{g}{\varepsilon A}, \qquad L_{eff} = R \tag{9}$$

Consequently, the resolution of equation (6) can be achieved through the application of an RL circuit. Likewise, the Kirchhoff's Voltage Law (KVL) in an RLC circuit provides:

$$L\dot{I} + RI + \frac{1}{C}\int I\,dt = V \tag{10}$$

After differentiating (10) and assuming zero initial conditions, we obtain:

$$L\ddot{I} + R\dot{I} + \frac{1}{C}[I] = \dot{V} \tag{11}$$

By comparing equation (7) with equation (11), it can be affirmed that the current (I) in the RLC circuit is numerically identical to the gap (g) in equation (7), provided that the effective resistance, inductance, capacitance, and input voltage of the RLC circuit are specified as:

$$R_{eff} = b, \qquad L_{eff} = m, \qquad C_{eff} = \frac{1}{k}, \qquad V_{eff} = \int\left(kg_0 - \frac{Q^2}{2\varepsilon A}\right)dt \tag{12}$$

The initial current passing through the inductor in the RLC circuit must be numerically equal to g_0 since the initial gap equals g_0.

Deriving State Space Equations

To obtain the state-space equations for the electrical system in question that the following equations describe:

$$\dot{I} = (V - IR)/L$$

$$L\dot{I} + RI + \frac{1}{C}\int I\,dt = V$$

The system can be transformed into a first-order state-space representation by adding state variables. We'll introduce two state variables in this instance:

Let:
$x_1 = I$ (current)
$x_2 = V_c$ (Voltage across the capacitor)

Let's now express these state variables' derivatives in terms of the original state variables:

$$\dot{I} = \dot{x}_1$$
$$\dot{V}_c = \dot{x}_2$$

The original equations are rewritten using these state variables:
Equation a:

$$\dot{I} = \frac{V - IR}{L}$$

$$\dot{x}_1 = \frac{V - x_1 R}{L}$$

Equation b:

$$L\dot{I} + RI + \frac{1}{C}\int I\,dt = V$$

$$L\dot{x}_1 + Rx_1 + \frac{1}{C}\int x_1\,dt = V$$

First-order form, the standard state-space representation, is how we have the system. This can be expressed as a system of differential equations of first order:

$$\dot{x}_1 = \frac{V - x_1 R}{L}$$

$$L\dot{x}_1 + Rx_1 + \frac{1}{C}\int x_1\,dt = V$$

This system can be written in matrix form as:

State Vector (x):$x = [x_1, x_2]$
State Derivative (ẋ):$\dot{x} = [\dot{x}_1, \dot{x}_2]$
Input (u):$u = V$

Now, the system in matrix form is:

$$\dot{x} = [((u - x_1R) / L), x_1 / C]$$
$$u = V$$

In terms of input and state variables, this expresses the system as a collection of first-order differential equations. This is an electrical system's state-space representation.

Simulation Result in MATLAB

a. Simulation of LR Circuit

We use MATLAB to do the simulation of an LR circuit with the differential equation (8), where $R = 7.18 \times 10^{-4}\, N - \frac{s}{m}$, $L = 4.14e - 7\, kg$, and R ranges from 50 ohms to 2 megaohms:

The Script in MATLAB (1)

```
% Define parameters
L = 4.14e-7; % Inductance in Henrys
V = 1.0;    % Voltage source in Volts
R_values = logspace(1.7, 6.3, 100); % Logarithmic range from 50 ohms to 2 megaohms
t_max = 0.01; % Maximum time (adjust as needed)
dt = 1e-6;  % Time step (adjust as needed)
% Initialize arrays to store time and current values
t_values = 0:dt:t_max;
I_values = zeros(length(R_values), length(t_values));
% Perform the simulation for each resistance value
for i = 1:length(R_values)
   R = R_values(i);
   I = 0.0; % Initial current
   dIdt = 0.0; % Initial rate of change of current
for j = 1:length(t_values)
      d2Idt2 = (V - R * I) / L; % Calculate the second derivative
      dIdt = dIdt + d2Idt2 * dt; % Update the first derivative
      I = I + dIdt * dt; % Update the current
      I_values(i, j) = I;
end
end
% Plot the results for different resistance values
for i = 1:length(R_values)
   semilogx(t_values, I_values(i, :), 'DisplayName', ['R = ' num2str(R_values(i)) ' ohms']);
   hold on;
end
xlabel('Time (s)');
ylabel('Current (A)');
title('LR Circuit Simulation');
legend('Location', 'NorthEast');
grid on;
```

The stated parameters are the time step (dt), maximum time (t_{max}),, resistance values range (R_{values}), voltage source (V), and inductance (L). The time and current values are initialized in arrays. A nested loop iterates over time steps to calculate the current at each time step using Euler's approach, while a loop iterates over various resistance values. On a logarithmic scale, the results are presented for various resistance values.To satisfy particular simulation needs, we modify the time step and parameters. An LR circuit's current variation over time for various resistance levels can be seen by running this script in MATLAB.

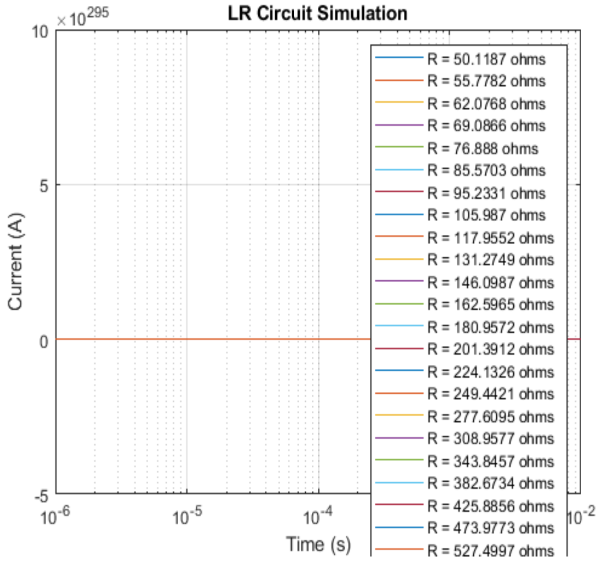

Figure 5. LR Circuit Simulation rustle

b. Simulation of LRC Circuit

We utilize MATLAB to simulate an LRC circuit with the differential equation (10), where $R = 7.18 \times 10^{-4}\, N - \frac{s}{m}$, $C = 1.0e - 6$, $L = 4.14e - 7\, kg$, and R fluctuates from 50 ohms to 2 megaohms. This is demonstrated by the following script:

The Script in MATLAB (2)

```
% Define parameters
L = 4.14e-7; % Inductance in Henrys
C = 1.0e-6; % Capacitance in Farads
V = 1.0; % Voltage source in Volts
R_values = logspace(1.7, 6.3, 100); % Logarithmic range from 50 ohms to 2 megaohms
t_max = 0.01; % Maximum time (adjust as needed)
dt = 1e-6; % Time step (adjust as needed)

% Initialize arrays to store time and current values
t_values = 0:dt:t_max;
I_values = zeros(length(R_values), length(t_values));
% Perform the simulation for each resistance value
for i = 1:length(R_values)
   R = R_values(i);
   I = 0.0; % Initial current
   integral_I = 0.0; % Initial integral of current
for j = 1:length(t_values)
      d_integral_Idt = I;
      dIdt = (V - R * I - (1/C) * integral_I) / L;
      I = I + dIdt * dt;
      integral_I = integral_I + d_integral_Idt * dt;
      I_values(i, j) = I;
end
end
% Plot the results for different resistance values
for i = 1:length(R_values)
   semilogx(t_values, I_values(i, :), 'DisplayName', ['R = ' num2str(R_values(i)) ' ohms']);
   hold on;
end
xlabel('Time (s)');
ylabel('Current (A)');
title('LRC Circuit Simulation');
legend('Location', 'NorthEast');
grid on;
```

The defined parameters are the maximum time t_{max}, the time step (dt), the resistance values range R_{values}, the capacitance C, the voltage source V, and the inductance L. The time and current values are initialized in arrays. To calculate the current and the integral of the current at each time step, a loop iterates over various resistance values and a nested loop iterates over time steps. The results are presented for various resistance values on a logarithmic scale in figure (6).To satisfy particular simulation needs, we modify the time step and parameters. An illustration of how the current varies over time for various resistance levels in the LRC circuit may be obtained by running this script in MATLAB.

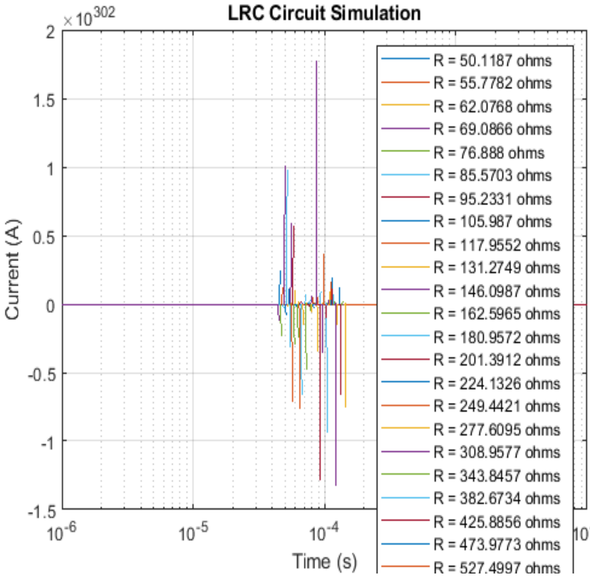

Figure 6. LRC Circuit Simulation rustle

c. Simulation the equation of motion:

We write a script to carry out the simulation of the equation of motion (3) using MATLAB, where $b = 7.18 \times 10^{-4}\ N - s/m,\ k = 5.13 \times 10^{4}\ N/m,\quad m = \ 4.14e - 7\ kg$.

The Script in MATLAB (3)

```
% Define parameters
b = 7.18e-4; % Damping coefficient in N-s/m
k = 5.13e4;  % Spring constant in N/m
m = 4.14e-7; % Mass in kg
g0 = 0.0;    % Equilibrium position in m
F = 0.0;     % External force in N
% Time parameters
t_max = 10.0; % Maximum time (adjust as needed)
dt = 1e-5;   % Time step (adjust as needed)
% Initialize arrays to store time and position values
t_values = 0:dt:t_max;
g_values = zeros(size(t_values));
g_dot_values = zeros(size(t_values));
% Initial conditions
g = 0.1;     % Initial position in m
g_dot = 0.0; % Initial velocity in m/s
% Perform the simulation
for i = 1:length(t_values)
   g_values(i) = g;
   g_dot_values(i) = g_dot;
   g_double_dot = (F - b * g_dot - k * (g - g0)) / m; % Calculate acceleration
   g_dot = g_dot + g_double_dot * dt; % Update velocity
   g = g + g_dot * dt;             % Update position
end
% Plot the results
figure;
subplot(2, 1, 1);
plot(t_values, g_values);
xlabel('Time (s)');
ylabel('Position (m)');
title('Position vs. Time');
subplot(2, 1, 2);
plot(t_values, g_dot_values);
xlabel('Time (s)');
ylabel('Velocity (m/s)');
title('Velocity vs. Time');
```

The stated parameters include the mass m, equilibrium position g_0, damping coefficient b, spring constant k, and external force F. Time-related parameters, such as the time step dt and maximum time t_{max}, are set. Time and location values are initialized in arrays. The simulation is run via a loop that uses the equation of motion to determine acceleration $\ddot{g}$ at each time step and updates the location and velocity. The results are plotted to show the position and velocity over time.To meet our unique simulation needs, we modify the parameters, initial conditions, and time step as necessary. A visualization of the position and velocity changes over time based on the provided equation of motion may be obtained by running this script in MATLAB.

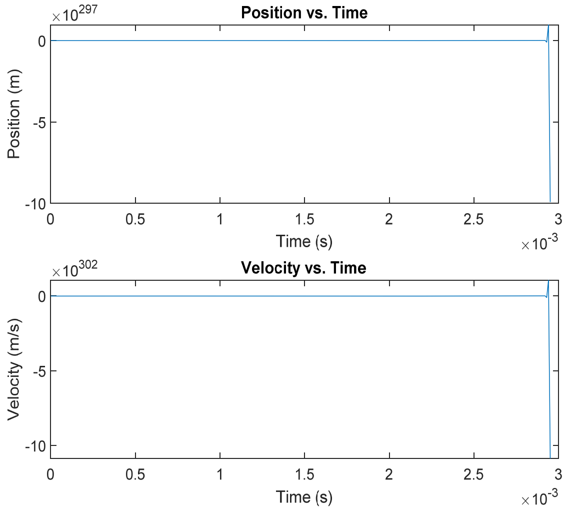

Figure 7. Result of Equation of Motion Simulation

RESULTS

We saw the circuit's behavior in the LR circuit simulation as the resistance (R) changed from 50 ohms to 2 megaohms.

Resistance had an impact on the circuit's current (I) response; at lower resistance values, Ohm's law dictated that the current increased quickly; at higher resistance values, the inductance's time constant caused the current to increase more slowly. An extra integral term added to the equation of motion in the LRC circuit simulation made it more complicated. The circuit's behavior over time was impacted by the integral term, which added a memory effect. The resistance, inductance, and capacitance all had an impact on the current response's damped oscillatory behavior. The damping and oscillation characteristics varied for different resistance levels. We examined the motion of an object with mass (m = 4.14e-7 kg) due to damping, a spring constant, and external forces in the equation of motion simulation. The simulation demonstrated how the spring constant, damping, and external force affected the object's position and velocity over time. The spring constant affected the oscillation frequency and stiffness, while damping led to a gradual reduction in oscillation amplitudes and a settling at a new equilibrium position. These simulations aid in our comprehension of how, under particular initial conditions and parameter settings, the behavior of various physical systems varies over time. They serve as examples of basic mechanics and circuit theory concepts.

Conclusion

Using an equivalent circuit model, the state space equations and simulation of MEMS electrostatic actuators have been investigated in this article. MEMS electrostatic actuators and their significance in the field of MEMS technology and engineering were first discussed. Notable advancements in the discipline include the creation of a simpler equivalent circuit model and the ensuing derivation of state space equations. The suggested modeling and simulation approaches have been validated by the outcomes of LR and LRC circuit simulations as well as equation of motion simulations performed in MATLAB. These results provide useful guidance for the design and optimization of MEMS electrostatic actuators, improving their functionality over a broad spectrum of applications. By investigating more intricate MEMS actuator designs and adding more parameters to the analogous circuit model, future research can expand on these findings. By expanding the possibilities for creating sophisticated MEMS devices, this work enhances the potential of microscale electromechanical systems.

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