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Research Article

AN IMPROVED CALIBRATED ESTIMATOR OF POPULATION MEAN IN STRATIFIED RANDOM SAMPLING

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Abstract

Calibration technique is becoming more important in sample survey. Calibration estimation helps in improving the estimates of population parameters by making use of auxiliary information. This paper proposed a new calibrated estimator for estimating the population mean in stratified random sampling with a set of new calibration constrains using known coefficient of variation of the auxiliary variable. A new improved calibration weights are derived by using constraints with coefficient of variation of the auxiliary variable in addition to the constraint utilize by Tracy et al. (2003). An empirical comparison of the suggested estimator is carried out by using an artificial population to compare the result of the proposed method.

Keywords: Calibration Estimation, Auxiliary variable, Coefficient of Variation, Stratified Random Sampling.

1. INTRODUCTION

Calibration estimation can be defined as a method of adjusting weight in survey sampling in order to estimate population parameters with the help of auxiliary information. The definition of calibration is given by Devill and Sarandal (1992) in the paper "Calibration estimators in survey sampling". There are many studies which make improved calibration techniques for estimating population parameters such as kim *et al.* (2007), Singh and Arnab (2011) etc. In the stratified random sampling, calibration approach is used to get optimum strata weights. Tracy *et al.* (2003) and Kim *et al.* (2007), Singh and Arnab (2011), Koyuncu and Kadilar (2013) etc. suggested new calibration estimators in the stratified random sampling. This work proposes a new calibration estimator for estimating population mean with the set of new calibration constraints by using coefficient of variation.

2. Notations defined in Calibration Estimation

Consider a finite population of size N be divided in to L homogenous subgroups called strata, with the h-th stratum containing Nh Units, where $h = 1,2, \dots \dots L$ and$\sum_{h=1}^{L} N_h = N$.

A sample of size n_h is drawn using simple random sampling without replacement (SRSWOR) from the hth stratum such that$\sum_{h=1}^{L} n_h = n$.

The sample and population mean of study and auxiliary variable are$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$ respectively. The sample and population mean of auxiliary variable are $\bar{x}_h = \sum_{i=1}^{n_h} \frac{x_{hi}}{n_h}$ and $\bar{X}_h = \sum_{i=1}^{N_h} \frac{X_{hi}}{N_h}$ respectively. The sample population variance of the auxiliary variable are $s_{hx}^2 = \sum_{i=1}^{n_h} \frac{(x_{hi}-\bar{x}_h)^2}{n_h-1}$ and $S_{hx}^2 = \sum_{i=1}^{N_h} \frac{(X_{hi}-\bar{X}_h)^2}{N_h-1}$ respectively. The sample and population coefficient of variation of the auxiliary variable are $c_{xh} = s_{hx}/\bar{x}_h$ and $C_{Xh} = \frac{S_{hx}}{\bar{X}_h}$respectively. Where s_{hx} and S_{hx} are the sample and population standard deviation respectively.

The classical unbiased estimator of population mean in stratified random sampling is given by$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ (2.1)
Where $W_h = \frac{N_h}{N}$ is the stratum weight, which are obtained through the minimization the distance function given in (2.2) subject to different calibration constraints.

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \qquad (2.2)$$

Where $\Omega_h$are the calibrated weights.

3. Some Existing Calibration Estimator For Stratified Sampling Population Mean

Different calibration estimators have been proposed in literature for estimating the population mean of a stratified sampling using the auxiliary variable. Some of the calibration estimators are reviewed below.

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3.1 Tracy et al. (2003)

The calibration estimator for the stratified random sampling is defined by Tracy *et al.* (2003) as
$\bar{y}_{st}(Tr) = \sum_{h=1}^{L} \Omega_h \bar{y}_h$ (3.1)

Where Ω_h are the weights minimizing the chi-square distance function

$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h}$ (3.2)

And the calibration constraints are given in equation (3.3) and (3.4)

$\sum_{h=1}^{L} \Omega_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h$ (3.3)
$\sum_{h=1}^{L} \Omega_h s_{hx}^2 = \sum_{h=1}^{L} W_h S_{hx}^2$ (3.4)
The proposed estimator by Tracy et al. (2003) is expressed as:

$\bar{y}_{st}(Tr) = \sum_{h=1}^{L} W_h y_h + \hat{\beta}_{1(Tr)}(\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)) + \hat{\beta}_{2(Tr)}(\sum_{h=1}^{L} W_h(S_{hx}^2 - s_{hx}^2))$ (3.5)

Where,

$$\hat{\beta}_{1(Tr)} = \frac{(\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h) - (\sum_{h=1}^{L} Q_h W_h \bar{x}_h s_{hx}^2)(\sum_{\{h=1\}}^{L} W_h Q_h s_{hx}^2 \bar{y}_h)}{(\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} Q_h W_h \bar{x}_h^2) - (\sum_{h=1}^{L} Q_h W_h \bar{x}_h s_{hx}^2)^2}$$

$$\hat{\beta}_{2(Tr)} = \frac{(\sum_{h=1}^{L} Q_h W_h \bar{x}_h^2)(\sum_{h=1}^{L} W_h Q_h s_{hx}^2 \bar{y}_h) - (\sum_{h=1}^{L} Q_h W_h \bar{x}_h s_{hx}^2)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h)}{(\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} Q_h W_h \bar{x}_h^2) - (\sum_{h=1}^{L} Q_h W_h \bar{x}_h s_{hx}^2)^2}$$

3.2 Nidhi et al.(2007)

The basic unbiased estimator of stratified random sampling is defined as

$\bar{y}_{st(N)} = \sum_{h=1}^{L} \Omega_h \bar{y}_h$ (3.6)

Where Ω_h are the weights minimizing the chi-square distance function

$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h}$ (3.7)

And the calibration constraints are given in equation (3.8) and (3.9)

$\sum_{h=1}^{L} \Omega_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h$ (3.8)
$\sum_{h=1}^{L} \Omega_h = 1$ (3.9)

The proposed estimator by Nidhi et al. (2007) is expressed as:

$$\bar{y}_{st(N)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_N \left(\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)\right)$$

Where,

$$\hat{\beta}_N = \left[\frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h)(\sum_{h=1}^{L} W_h Q_h \bar{y}_h)}{(\sum_{h=1}^{L} W_h Q_h \bar{x}_{hx}^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h)^2}\right]$$

3.3 Nursel Koyuncu et al. (2015)

They introduced a new calibration estimator in stratified random sampling which is given as
$\bar{y}_{st(Nk)} = \sum_{h=1}^{L} \Psi_h \bar{y}_h$ (3.10)

Where Ψ_h are new calibration weights minimizing chi-square distance function

$\sum_{h=1}^{L} \frac{(\Psi_h - W_h)^2}{Q_h W_h}$ (3.11)

And the calibration constraints are given in equation (3.12), (3.13) and (3.14)

$\sum_{h=1}^{L} \Psi_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h$ (3.12)
$\sum_{h=1}^{L} \Psi_h s_{hx}^2 = \sum_{h=1}^{L} W_h S_{hx}^2$ (3.13)
$\sum_{h=1}^{L} \Psi_h = \sum_{h=1}^{L} W_h$ (3.14)

The proposed calibrated estimator by NurselKoyuncu (2015) is expressed as

$$\bar{y}_{st(Nk)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{1(Nk)}[\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)] + \hat{\beta}_{2(Nk)}[\sum_{h=1}^{L} W_h(S_{hx}^2 - s_{hx}^2)]$$

Where betas are given by

$$\hat{\beta}_{1(Nk)} = \frac{A_4}{B}, \hat{\beta}_{2(Nk)} = \frac{A_5}{B}$$

Where,

$$\begin{aligned} A_4 = &\left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^4\right) - \left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2\right)^2\right] \\ &- \left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{x}_h\right)\left(\sum_{h=1}^{L} Q_h W_h\right) - \left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2\right)\right] \\ &+ \left(\sum_{h=1}^{L} Q_h W_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{x}_h\right) - \left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^4\right)\right] \end{aligned}$$

$$\begin{aligned} A_5 = &\left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2\right) - \left(\sum_{h=1}^{L} Q_h W_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{x}_h\right)\right] \\ &+ \left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h\right)\left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h^2\right) - \left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h\right)^2\right] \\ &+ \left(\sum_{h=1}^{L} Q_h W_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2 \bar{x}_h\right) - \left(\sum_{h=1}^{L} Q_h W_h \bar{x}_h^2\right)\left(\sum_{h=1}^{L} Q_h W_h s_{hx}^2\right)\right] \end{aligned}$$

3.4 Rao et al. (2016)

They developed two different calibration estimators given as

$\bar{y}_{st(R1)} = \sum_{h=1}^{L} \Omega_{h1} \bar{y}_h$ (3.15)
$\bar{y}_{st(R2)} = \sum_{h=1}^{L} \Omega_{h2} \bar{y}_h$ (3.16)

Where Ω_{h1} and Ω_{h2} are the new weights obtained by minimizing the chi square distance function

$\sum_{h=1}^{L} \frac{(\Omega_{h1} - W_h)^2}{Q_h W_h}$ (3.17)

$\sum_{h=1}^{L} \frac{(\Omega_{h2} - W_h)^2}{Q_h W_h}$ (3.18)

Andthe calibration constraints are given in equation (3.19) and (3.20)

$\sum_{h=1}^{L} \Omega_{h1}(\bar{x}_h + c_{hx}) = \sum_{h=1}^{L} W_h(\bar{X}_h + C_{hx})$ (3.19)
$\sum_{h=1}^{L} \Omega_{h2}(1 + \bar{x}_h + c_{hx}) = \sum_{h=1}^{L} W_h(1 + \bar{X}_h + C_{hx})$ (3.20)

The two calibrated estimator proposed by Rao et al. (2016) are:

$$\bar{y}_{st(R1)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{1(R)}[\sum_{h=1}^{L} W_h(\bar{X}_h + C_{hx}) - \sum_{h=1}^{L} W_h(\bar{x}_h + c_{hx})$$

$$\bar{y}_{st(R2)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{2(R)}[\sum_{h=1}^{L} W_h(1 + \bar{X}_h + C_{hx}) - \sum_{h=1}^{L} W_h(1 + \bar{x}_h + c_{hx})$$

Where,

$$\hat{\beta}_{1(R)} = \left[\frac{\sum_{h=1}^{L} W_h Q_h \bar{y}_h(\bar{x}_h + c_{hx})}{\sum_{h=1}^{L} W_h Q_h(\bar{x}_h + c_{hx})^2}\right]$$

$$\hat{\beta}_{2(R)} = \left[\frac{\sum_{h=1}^{L} W_h Q_h \bar{y}_h (1+\bar{x}_h + c_{hx})}{\sum_{h=1}^{L} W_h Q_h (1+\bar{x}_h + c_{hx})^2}\right]$$

3.5 Sisodia et al. (2017)

They introduced a calibration estimator is given as

$$\bar{y}_{st(S)} = \sum_{h=1}^{L} \Omega_h \bar{y}_h \quad (3.21)$$

Where Ω_h are the new weights obtained by minimizing the chi square distance function

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \quad (3.22)$$

And the calibration constraints are given in equation (3.23) and (3.24)

$$\sum_{h=1}^{L} W_h = 1 \quad (3.23)$$
$$\sum_{h=1}^{L} \Omega_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h \quad (3.24)$$

The calibrated estimator proposed by Sisodia et al. (2017) is expressed as:

$$\bar{y}_S = \sum_{h=1}^{L} W_h [\bar{y}_h + \hat{\beta}_S (\bar{X}_h - \bar{x}_h)]$$

Where,

$$\hat{\beta}_S = \left[\frac{(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h)}{(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h)^2}\right]$$

3.6 Ozqul (2018)

They developed a calibrated estimator is given as

$$\bar{y}_{st(O)} = \sum_{h=1}^{L} \Omega_h \bar{y}_h \quad (3.25)$$

Where Ω_h are the new weights obtained by minimizing the chi square distance function

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \quad (3.26)$$

And the calibration constraints are given in equation (3.27),(3.28) and (3.29)

$$\sum_{h=1}^{L} W_h = 1 \quad (3.27)$$
$$\sum_{h=1}^{L} \Omega_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h \quad (3.28)$$
$$\sum_{h=1}^{L} \Omega_h c_{hx} = \sum_{h=1}^{L} W_h C_{hx} \quad (3.29)$$

The proposed estimator by Ozqul (2018) is expressed as:

$$\bar{y}_{st(O)} = \sum_{h=1}^{L} [\bar{y}_h + \hat{\beta}_{1(O)} (\bar{X}_h - \bar{x}_h) + \hat{\beta}_{2(O)} (C_{hx} - c_{hx})]$$

Where,

$$\hat{\beta}_{1(O)} = \frac{\gamma_1}{\eta} \text{and} \hat{\beta}_{2(O)} = \frac{\gamma_2}{\eta}$$

$$\begin{aligned}\gamma_1 = &\left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}^2\right)\right] \\ &+ \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right)^2\right] \\ &+ \left(\sum_{h=1}^{L} W_h Q_h c_{hx} \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right) - \left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right)\right]\end{aligned}$$

$$\gamma_2 = \left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)\right] + \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right)\left(\sum_{h=1}^{L} W_h Q_h\right)\right] + \left(\sum_{h=1}^{L} W_h Q_h c_{hx} \bar{y}_h\right)\left[\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)^2\right]$$

$$\eta = \left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right)^2 - \left(\sum_{h=1}^{L} W_h Q_h c_{hx}^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)^2 + 2\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{hx}\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_h c_{hx}\right)$$

3.7 Alam et al. (2019)

The calibration estimator of finite population mean is defined as

$$\bar{y}_{st(A1)} = \sum_{h=1}^{L} W_h^{(1)} \bar{y}_h \quad (3.30)$$

Where $W_h^{(1)}$ is the new calibrated weight obtained by minimizing the new distance function

$$\frac{1}{2}\sum_{h=1}^{L} \frac{\left(W_h^{(1)} - W_h\right)^2}{W_h Q_h} + \sum_{h=1}^{L} \sum_{h \neq h'=1}^{L} \left(W_h^{(1)} - W_h\right)\left(W_{h'}^{(1)} - W_{h'}\right) \quad (3.31)$$

The calibration constraints is given in equation (3.32)

$$\sum_{h=1}^{L} W_h^{(1)} \bar{x}_h = \bar{X} \quad (3.32)$$

The first proposed calibrated estimator is given as

$$\bar{y}_{st(A1)} = \sum_{h=1}^{L} w_h \bar{y}_h + \hat{\beta}_{1(A1)}\left(\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right)$$

Where,

$$\hat{\beta}_{1(A1)} = \frac{\sum_{h=1}^{L} \frac{(W_h Q_h \bar{x}_h \bar{y}_h)}{1 - W_h Q_h}}{\sum_{h=1}^{L} \frac{(W_h Q_h \bar{x}_h^2)}{1 - W_h Q_h}}$$

Similarly, the second calibration estimator of finite population mean is defined as

$$\bar{y}_{st(A2)} = \sum_{h=1}^{L} W_h^{(2)} \bar{y}_h \quad (3.33)$$

Where $W_h^{(2)}$ is the new calibrated weight obtained by minimizing the new distance function

$$\frac{1}{2}\sum_{h=1}^{L} \frac{\left(W_h^{(2)} - W_h\right)^2}{W_h Q_h} + \sum_{h=1}^{L} \sum_{h \neq h'=1}^{L} \left(W_h^{(2)} - W_h\right)\left(W_{h'}^{(2)} - W_{h'}\right) \quad (3.34)$$

The calibration constraints is given in equation (3.35) and (3.36)

$$\sum_{h=1}^{L} W_h^{(2)} = \sum_{h=1}^{L} W_h \quad (3.35)$$
$$\sum_{h=1}^{L} W_h^{(2)} \bar{x}_h = \bar{X} \quad (3.36)$$

The second calibrated estimator of finite population mean is defined as

$$\bar{y}_{st(A2)} = \sum_{h=1}^{L} w_h \bar{y}_h + \hat{\beta}_{2(A2)}\left(\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right)$$

Where,

$$\hat{\beta}_{2(A2)} = \frac{\sum_{h=1}^{L}\left(\frac{W_h Q_h \bar{x}_h \bar{y}_h}{1 - W_h Q_h}\right)\sum_{h=1}^{L}\left(\frac{W_h Q_h}{1 - W_h Q_h}\right) - \sum_{h=1}^{L}\left(\frac{W_h Q_h \bar{x}_h}{1 - W_h Q_h}\right)\sum_{h=1}^{L}\left(\frac{W_h Q_h \bar{y}_h}{1 - W_h Q_h}\right)}{\sum_{h=1}^{L}\left(\frac{W_h Q_h}{1 - W_h Q_h}\right)\sum_{h=1}^{L}\left(\frac{W_h Q_h \bar{x}_h^2}{1 - W_h Q_h}\right) - \left(\sum_{h=1}^{L}\left(\frac{W_h Q_h \bar{x}_h}{1 - W_h Q_h}\right)\right)^2}$$

3.8 Garg and Pachori (2019)

The developed calibration estimator expressed as

$$\bar{y}_{st(G)} = \sum_{h=1}^{L} \Omega_h \, \bar{y}_h \tag{3.37}$$

Where Ω_h are the new weights obtained by minimizing the chi square distance function

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \tag{3.38}$$

The calibration constraints are given in equation (3.39) and (3.40)

$$\sum_{h=1}^{L} \Omega_h = \sum_{h=1}^{L} W_h (3.39)$$

$$\sum_{h=1}^{L} \Omega_h \, c_{hx} = \sum_{h=1}^{L} W_h \, C_{hx} \tag{3.40}$$

The proposed estimator by Garg and Pachori (2019) is expressed as:

$$\bar{y}_{st(G)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{(G)} \left[\sum_{h=1}^{L} W_h (C_{hx} - c_{hx}) \right]$$

Where,

$$\hat{\beta}_{(G)} = \frac{(\sum_{h=1}^{L} W_h Q_h)(\sum_{h=1}^{L} W_h Q_h c_{hx} \bar{y}_h) - (\sum_{h=1}^{L} W_h Q_h c_{hx})(\sum_{h=1}^{L} W_h Q_h \bar{y}_h)}{(\sum_{h=1}^{L} W_h Q_h c_{hx}^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h c_{hx})^2}$$

3.9 Alam and Shabbir (2020)

The basic unbiased estimator of stratified random sampling is defined as

$$\bar{y}_{st(AS)} = \sum_{h=1}^{L} \Omega_h \, \bar{y}_h \tag{3.41}$$

Where Ω_h are the weights minimizing the chi-square distance function

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \tag{3.42}$$

The calibration constraints are given in equation (3.43) and (3.44)

$$\sum_{h=1}^{L} \Omega_h \bar{x}_h = \sum_{h=1}^{L} W_h \bar{X}_h \tag{3.43}$$

$$\sum_{h=1}^{L} \Omega_h \bar{r}(x)_h = \sum_{h=1}^{L} W_h \bar{R}(x)_h \tag{3.44}$$

The proposed estimator by Alam and Shabbir(2020) is expressed as

$$\bar{y}_{st(AS)} = \sum_{h=1}^{L} W_h \left[\bar{y}_h + \hat{\beta}_{1(AS)} (\bar{X}_h - \bar{x}_h) + \hat{\beta}_{2(AS)} (\bar{R}(x)_h - \bar{r}(x)_h) \right]$$

Where,

$$\hat{\beta}_{1(AS)} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h \bar{r}(x)_h^2) - (\sum_{h=1}^{L} W_h Q_h \bar{y}_h \, \bar{r}(x)_h)(\sum_{h=1}^{L} W_h Q_h \bar{r}(x)_h)}{(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2)(\sum_{h=1}^{L} W_h Q_h \bar{r}(x)_h^2) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h \, \bar{r}(x)_h)^2} \right]$$

$$\hat{\beta}_{2(AS)} = \left[\frac{(\sum_{h=1}^{L} W_h Q_h \bar{y}_h \, \bar{r}(x)_h)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h \bar{x}_h \, \bar{r}(x)_h)}{(\sum_{h=1}^{L} W_h Q_h \bar{x}_h^2)(\sum_{h=1}^{L} W_h Q_h \bar{r}(x)_h^2) - (\sum_{h=1}^{L} W_h Q_h \bar{x}_h \, \bar{r}(x)_h)^2} \right]$$

3.10 Oluwagbenga T. Babatunde et Al. (2022)

They introduced a calibration estimator for population mean of a stratified random sampling in the presence of outlier in the auxiliary variable is given as

$$\bar{y}_{st(M)} = \sum_{h=1}^{L} \Omega_h \, \bar{y}_h \tag{3.45}$$

Where Ω_h are the weights minimizing the chi-square distance function

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \tag{3.46}$$

The calibration constraints are given in equation (3.47) and (3.48)

$$\sum_{h=1}^{L} \Omega_h = \sum_{h=1}^{L} W_h \quad (3.47)$$
$$\sum_{h=1}^{L} \Omega_h m_h = \sum_{h=1}^{L} W_h M_h \quad (3.48)$$

The proposed calibrated estimator is given as

$$\bar{y}_{st(M)} = \sum_{h=1}^{L} W_h\left[\bar{y}_h + \hat{\beta}_M(M_h - m_n)\right]$$

Where,

$$\hat{\beta}_M = \left[\frac{(\sum_{h=1}^{L} W_h Q_h m_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h \bar{y}_h)(\sum_{h=1}^{L} W_h Q_h m_h)}{(\sum_{h=1}^{L} W_h Q_h m_h^2)(\sum_{h=1}^{L} W_h Q_h) - (\sum_{h=1}^{L} W_h Q_h m_h)^2}\right]$$

4. The proposed estimator

Following Tracy *et al.* (2003), we introduced a new calibration estimator as given by

$$\bar{y}_{st}(Nr) = \sum_{h=1}^{L} \Omega_h \bar{y}_h \quad (4.1)$$

Where the calibrated weights $\Omega_h,\ h = 1,2,\cdots,L$ are chosen to minimize the chi-square distance function given as

$$\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} \quad (4.2)$$

To get the reasonable weights we have consider following calibration constraints in stratified random sampling:

$$\sum_{h=1}^{L} \Omega_h c_{xh} = \sum_{h=1}^{L} W_h C_{Xh} \quad (4.3)$$

$$\sum_{h=1}^{L} \Omega_h s_{hx}^2 = \sum_{h=1}^{L} W_h S_{hx}^2 \quad (4.4)$$

Here $c_{xh} = s_{hx}/\bar{x}_h$ and $C_{Xh} = S_{hx}/\bar{X}_h$ are the h-thstratum sample and population coefficient of variation of the auxiliary variable, respectively.

The langrange function is given as

$$L = \sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{Q_h W_h} - 2\lambda_1\left(\sum_{h=1}^{L} \Omega_h c_{xh} - \sum_{h=1}^{L} W_h C_{Xh}\right) - 2\lambda_2\left(\sum_{h=1}^{L} \Omega_h s_{hx}^2 - \sum_{h=1}^{L} W_h S_{hx}^2\right) \quad (4.5)$$

Where λ_1 and λ_2 are the langrange's multiplier to find the optimum value of Ω_h.

Now, differentiate the eq. (4.5) with respect to Ω_h and equate to zero.

$$\frac{\partial \Delta}{\partial \Omega_h} = \frac{2(\Omega_h - W_h)}{Q_h W_h} - 2\lambda_1 c_{xh} - 2\lambda_2 s_{hx}^2$$

$$\frac{(\Omega_h - W_h)}{Q_h W_h} - \lambda_1 c_{xh} - \lambda_2 s_{hx}^2 = 0$$

$$\Omega_h = W_h + Q_h W_h(\lambda_1 c_{xh} + \lambda_2 s_{hx}^2) \quad (4.6)$$

Substituting the weights Ω_h in (4.3) and (4.4),we have

$$\lambda_1 \sum_{h=1}^{L} W_h Q_h c_{xh}^2 + \lambda_2 \sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2 = \sum_{h=1}^{L} W_h C_{Xh} - \sum_{h=1}^{L} W_h c_{xh} \quad (4.7)$$
$$\lambda_1 \sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2 + \lambda_2 \sum_{h=1}^{L} W_h Q_h s_{hx}^4 = \sum_{h=1}^{L} W_h S_{hx}^2 - \sum_{h=1}^{L} W_h s_{hx}^2 \quad (4.8)$$

Now from eq. (4.7) and (4.8)

$$\begin{bmatrix} \sum_{h=1}^{L} W_h Q_h c_{xh}^2 & \sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2 \\ \sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2 & \sum_{h=1}^{L} W_h Q_h s_{hx}^4 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \sum_{h=1}^{L} W_h C_{Xh} - \sum_{h=1}^{L} W_h c_{xh} \\ \sum_{h=1}^{L} W_h S_{hx}^2 - \sum_{h=1}^{L} W_h s_{hx}^2 \end{bmatrix} \quad (4.9)$$

$$\lambda_1 = \frac{[(\sum_{h=1}^{L} W_h Q_h s_{hx}^4)(\sum_{h=1}^{L} W_h (C_{Xh} - c_{xh})) - (\sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2)(\sum_{h=1}^{L} W_h (S_{hx}^2 - s_{hx}^2))]}{[(\sum_{h=1}^{L} W_h Q_h c_{xh}^2)(\sum_{h=1}^{L} W_h Q_h c_{xh}^2) - (\sum_{h=1}^{L} (W_h Q_h c_{xh} s_{hx}^2)^2)]}$$

$$\lambda_2 = \frac{[(\sum_{h=1}^{L} W_h Q_h c_{xh}^2)(\sum_{h=1}^{L} W_h (S_{hx}^2 - s_{hx}^2)) - (\sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2)(\sum_{h=1}^{L} W_h (C_{Xh} - c_{xh}))]}{[(\sum_{h=1}^{L} W_h Q_h c_{xh} s_{hx}^2)(\sum_{h=1}^{L} W_h Q_h s_{hx}^4) - (\sum_{h=1}^{L} (W_h Q_h c_{xh} s_{hx}^2)^2)]}$$

Substituting the weightsΩ_h, the estimators (4.1) is

$$\bar{y}_{st}(Nr) = \sum_{\{h=1\}}^{L} W_h y_h + \hat{\beta}_{1(Nr)} \left(\sum_{h=1}^{L} W_h (C_{Xh} - c_{xh}) \right) + \hat{\beta}_{2(Nr)} \left(\sum_{h=1}^{L} W_h (S_{hx}^2 - s_{hx}^2) \right)$$

Where,

$$\hat{\beta}_{1(Nr)} = \frac{(\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} W_h Q_h c_{xh} \bar{y}_h) - (\sum_{h=1}^{L} Q_h W_h c_{xh} s_{hx}^2)(\sum_{h=1}^{L} W_h Q_h s_{hx}^2 \bar{y}_h)}{\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} Q_h W_h c_{xh}^2) - (\sum_{h=1}^{L} Q_h W_h c_{xh} s_{hx}^2)^2}$$

$$\hat{\beta}_{2(Nr)} = \frac{(\sum_{h=1}^{L} Q_h W_h c_{xh}^2)(\sum_{h=1}^{L} W_h Q_h s_{hx}^2 \bar{y}_h) - (\sum_{h=1}^{L} Q_h W_h c_{xh} s_{hx}^2)(\sum_{h=1}^{L} W_h Q_h c_{xh} \bar{y}_h)}{\sum_{h=1}^{L} Q_h W_h s_{hx}^4)(\sum_{h=1}^{L} Q_h W_h c_{xh}^2) - (\sum_{h=1}^{L} Q_h W_h c_{xh} s_{hx}^2)^2}$$

5. Simulation Study

To study the performance of the proposed estimator a limited simulation study has been carried out with four different artificial generated populations. The population given in Tracy *et al.* (2003) and Nurselkoyuncu(2015). Where X_{hi} and y_{hi} values are form different distributions.The populations given in Table. 1(Appendix 1).

The correlation coefficients between study and auxiliary variables for each stratum taken as $\rho_{xy1} = 0.5$, $\rho_{xy2} = 0.7, \rho_{xy3} = 0.9$respectively. The quantities $S_{1x} = 4.5$,$S_{2x} = 6.2$,$S_{3x} = 8.4$andS_{1y} ,S_{2y} ,$S_{3y} = 4.8$ were fixed in each stratum. All four population consist three strata having 5 units, the sample of size $n_h = 2,3,4$ units are drawn from each stratum. This process have been repeated 500 time independently that is 500 samples of size 2,3,4 units have been drawn from each stratum from given population. The MSE of Tracy's estimator and the proposed estimator is given by the formula

$$MSE(\bar{y}_{st}(\alpha)) = \frac{\sum_{k=1}^{(500)} [\bar{y}_{st}(\alpha) - \bar{Y}]}{(500)}, \alpha = Tr, Nr$$

$$PRE = \frac{MSE(\bar{y}_{st}(Tr))}{MSE(\bar{y}_{st}(Nr))} * 100$$

Table 1. The PRE of proposed estimator in comparison of Tracy's *et al.*

Population	Empirical MSE $\bar{y}_{st}(Tr)$	Empirical MSE $\bar{y}_{st}(Nr)$	PRE
I	21325941	16773078	127.143
II	1319504	1043659	126.430
III	55131925	46055246	119.708
IV	9106657	7630231	119.349

Form the above Table 3 it is clear that the proposed estimator is more efficient then Tracy et al. (2003) for given generated populations**.**

6. Conclusion

In this work, a new calibrated estimator is proposed to estimate the population mean in stratified random sampling. A simulation study has been performed to compare the efficiency of the suggested estimator. In the simulation study, we consider four populations and calculate the estimators from all possible samples. The empirical study performed by using R code (Appendix 2). The result shows that the suggested estimator is more efficient than Tracy et al. (2003).

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APPENDIX

Appendix 1

Table 1. Parameters and distributions of study and auxiliary variables

Populations	Parameters and distributions of the study Variable	Parameters and distributions of the auxiliary variable
1. Population $h = 1,2,3$	$f(y^*_{hi}) = \frac{1}{\Gamma(1.5)} y_{hi}^{*1,5-1} e^{-y^*_{hi}}$	$f(x^*_{hi}) = \frac{1}{\Gamma(0.3)} x_{hi}^{*0,3-1} e^{-x^*_{hi}}$
2. Population $h = 1,2,3$	$f(y^*_{hi}) = \frac{1}{\Gamma(0.3)} y_{hi}^{*0.3-1} e^{-y^*_{hi}}$	$f(x^*_{hi}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^{*2}_{hi}}{2}}$
3. Population $h = 1,2,3$	$f(y^*_{hi}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^{*2}_{hi}}{2}}$	$f(x^*_{hi}) = \frac{1}{\Gamma(0.3)} x_{hi}^{*0,3-1} e^{-x^*_{hi}}$
4. Population $h = 1,2,3$	$f(y^*_{hi}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^{*2}_{hi}}{2}}$	$f(x^*_{hi}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^{*2}_{hi}}{2}}$

Table 2. Properties of strata

Strata	Study variable	Auxiliary variable
1. Stratum	$y_{1i} = 50 + y^*_{1i}$	$x_{1i} = 15 + \sqrt{(1-\rho^2_{xy1})x^*_{1i}} + \rho_{xy1}\left(\frac{S_{1x}}{S_{1y}}\right)y^*_{1i}$
2. Stratum	$y_{2i} = 150 + y^*_{2i}$	$x_{2i} = 100 + \sqrt{(1-\rho^2_{xy2})x^*_{2i}} + \rho_{xy2}\left(\frac{S_{2x}}{S_{2y}}\right)y^*_{2i}$
3. Stratum	$y_{3i} = 100 + y^*_{3i}$	$x_{3i} = 200 + \sqrt{(1-\rho^2_{xy3})x^*_{3i}} + \rho_{xy3}\left(\frac{S_{3x}}{S_{3y}}\right)y^*_{3i}$

Appendix 2

```
N1<-5;N2<-5;N3<-5;
N<-15
S1x<-4.5;S2x<-6.2;S3x<-8.4
S1y<-4.8;S2y<-4.8;S3y<-4.8
r1<-0.5;r2<-0.7;r3<-0.9
```

```
Q<-1;W1<-N1/N;W2<-N2/N;W3<-N3/N;
n<-c(2,3,4)
y1<-rgamma(N1,1.5,1)+50
y2<-rgamma(N2,1.5,1)+150
y3<-rgamma(N3,1.5,1)+100
x1<-15+sqrt(1-r1^2)*rgamma(N1,0.3,1)+r1*(S1x/S1y)*y1
x2<-100+sqrt(1-r2^2)*rgamma(N2,0.3,1)+r2*(S2x/S2y)*y2
x3<-200+sqrt(1-r3^2)*  rgamma(N3,0.3,1)+r3*(S3x/S3y)*y3
Mx1<-mean(x1);Mx2<-mean(x2);Mx3<-mean(x3)
My1<-mean(y1);My2<-mean(y2);My3<-mean(y3)
CX1<-S1x/mean(x1)
CX2<-S2x/mean(x2)
CX3<-S3x/mean(x3)
Mys<-W1*My1+W2*My2+W3*My3
dat1<-cbind(y1,x1);dat2<-cbind(y2,x2);dat3<-cbind(y3,x3)
y11<-NA;x12<-NA;y11s<-NA;x11s<-NA;x12s<-NA;Q1<-NA;Q2<-NA;Q3<-NA;Q4<-NA;Q5<-NA
Q6<-NA;Q7<-NA;Q8<-NA;Q9<-NA;b1<-NA;b2<-NA;b1nk<-NA;b2nk<-NA;A1<-NA;A2<-NA;b<-NA
y21<-NA;x22<-NA;x21s<-NA;x22s<-NA;y21s<-NA;MSE<-NA
y31<-NA;x32<-NA;x31s<-NA;x32s<-NA;y31s<-NA;maa1<-NA;maa2<-NA;maa3<-NA
ybart<-NA;MSEn<-NA
cv12<-NA;cv22<-NA;cv32<-NA;Q10<-NA;Q11<-NA;Q12<-NA;b1n<-NA;b2n<-NA;ybartcv<-NA;
for(i in 1 : 500){
m1<-c(sample(1:5,2,replace=F))
m2<-c(sample(1:5,3,replace=F))
m3<-c(sample(1:5,4,replace=F))
ma1=dat1[m1,];ma2=dat2[m2,];ma3=dat3[m3,]
maa1<-as.data.frame(ma1)
maa2<-as.data.frame(ma2)
maa3<-as.data.frame(ma3)
y11[i]<-mean(maa1$y1)
x12[i]<-mean(maa1$x1)
y11s[i]<-sd(maa1$y1)
x12s[i]<-sd(maa1$x1)
y21[i]<-mean(maa2$y2)
x22[i]<-mean(maa2$x2)
y21s[i]<-sd(maa2$y2)
x22s[i]<-sd(maa2$x2)
y31[i]<-mean(maa3$y3)
x32[i]<-mean(maa3$x3)
y31s[i]<-sd(maa3$y3)
x32s[i]<-sd(maa3$x3)
Q1[i]<-Q*W1*x12s[i]^4+Q*W2*x22s[i]^4+Q*W1*x32s[i]^4
Q2[i]<-Q*W1*y11[i]*x12[i]+Q*W2*y21[i]*x22[i]+Q*W3*y31[i]*x32[i]
Q3[i]<-Q*W1*x12s[i]^2*x12[i]+Q*W2*x22s[i]^2*x22[i]+Q*W3*x32s[i]^2*x32[i]
Q4[i]<-Q*W1*x12s[i]^2*y11[i]+Q*W2*x22s[i]^2*y21[i]+Q*W3*x32s[i]^2*y31[i]
Q5[i]<-Q*W1*x12[i]^2+Q*W2*x22[i]^2+Q*W3*x32[i]^2
b1[i]<-(Q1[i]*Q2[i]-Q3[i]*Q4[i])/(Q1[i]*Q5[i]-Q3[i]^2)
b2[i]<-(Q5[i]*Q4[i]-Q3[i]*Q2[i])/(Q1[i]*Q5[i]-Q3[i]^2)
ybart[i]<-W1*y11[i]+W2*y21[i]+W3*y31[i]+b1[i]*(W1*(Mx1-x12[i])
+(W2*(Mx2-x22[i]))+(W3*(Mx3-x32[i])))+b2[i]*(W1*(S1x^2-x12s[i]^2)
+W2*(S2x^2-x22s[i]^2)+W3*(S3x^2-x32s[i]^2))
MSE[i]<-(ybart[i]-Mys)^2

#. ..new estimator.....
cv12[i]<-x12s[i]/x12[i]
cv22[i]<-x22s[i]/x22[i]
cv32[i]<-x32s[i]/x32[i]
Q10[i]<-Q*W1*cv12[i]*y11[i]+Q*W2*cv22[i]*y21[i]+Q*W3*cv32[i]*y31[i]
Q11[i]<-Q*W1*cv12[i]*x12s[i]+Q*W2*cv22[i]*x22s[i]+Q*W3*cv32[i]*x32s[i]
Q12[i]<-Q*W1*cv12[i]^2+Q*W2*cv22[i]^2+Q*W3*cv32[i]^2
b1n[i]<-((Q10[i]*Q1[i]-Q4[i]*Q11[i])/(Q12[i]*Q1[i]-Q11[i]^2))
b2n[i]<-((Q4[i]*Q12[i]-Q10[i]*Q11[i])/(Q12[i]*Q1[i]-Q11[i]^2))
ybartcv[i]<-W1*y11[i]+W2*y21[i]+W3*y31[i]+b1n[i]*((W1*(CX1-cv12[i]))
+(W2*(CX2-cv22[i]))+(W3*(CX3-cv32[i])))+b2n[i]*((W1*(S1x^2-x12s[i]^2))
+(W2*(S2x^2-x22s[i]^2))+(W3*(S3x^2-x32s[i]^2)))
MSEn[i]<-(ybartcv[i]-Mys)^2
res<-mean(MSE);res
res2<-mean(MSEn);res2
pre<-(res/res2)*100;pre
pre1<-(res1/res2)*100;pre1
}
```

****