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Research Article

AN EXTENSION PROOF OF RIEMANN HYPOTHESIS BY A LOGICAL ENTAILS TRUTH TABLE

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Abstract

There were many mathematicians who tried to prove or disprove the statement of Riemann Hypothesis. However, none of them have been successfully approved by the Clay Mathematical Institute. In addition, to the best of this author's knowledge, these mathematicians haven't employed the technique of logical truth table during their proofs. With reference to this author's previous proof in [1], this author have employed the method of multiplicative telescope together with the prime boundary gaps. In this extended version of my proof to the Riemann Hypothesis, this author tries to show that RH statement is true through the four cases of the conditional statements in the truth table. Three of the cases (I, II, IV) are found to be true for the conditional statement in the Riemann Hypothesis while only one (case III) is found to be false (and acts as the disproof by a counter-example). Moreover, there are also three sub-cases (i, ii, iii) [1] among these four tabled cases. The main idea is that the we may disproof the hypothesis statement that is similar to the RH one by first find a counter-example which is obviously a disproof (case III) to the (Riemann) hypothesis. But it is NOT compatible with the GÖdel's Incompleteness Theorem. Otherwise either the disproof to the statement or the Gödel is incorrect which is impossible. Hence, the disproof is said to be incompatible with the Gödel. On the other hand, all of the other truth cases (I, II, IV) for the statement are indeed the examples for the positive results to the Riemann Hypothesis statement and are compatible with the Gödel. Therefore, the only way to make a conclusion is to say or force the Riemann Hypothesis statement to be correct. In general, for any hypothesis with the conditional statements structure like the Riemann one, we may also prove them by the similar techniqe and the arguments of the truth table for their conditional statements together with the Gödel's Incompleteness theorem to force the positive result for the hypothesis statement. Actually, there are many applications for the truth tables especially in the fields like language (structure & modeling) or in engineering (logic gates & programming) etc during our everyday usage.

Keywords: Quantum computing, Statement logic with branch, Riemann Hypothesis, Number theory and Taylor approximation model.

INTRODUCTION

As shown in my previous papers [1], this author have developed a suggested way to prove the long awaiting question for the truthness of the Riemann Hypothesis in a long historic period of time. In fact, with reference to [1], it employs the method of multiplative telescope together with some logics. Actually, one may extend [1] to prove the Riemann Hypothesis by the logical entails truth table in a novel and complete way. Then we may let the Riemann Hypothesis statement to be the hypothesis X and want to determine whether it may be correct or not. In fact, the truth table for the Riemann Hypothesis should like the following:

Hypothesis X	Consequent Y	Conditional X → Y
True (T)	True (T)	True (T)
False (F)	True (T)	True (T)
True (T)	False (F)	False (F)
False (F)	False (F)	True (T)

Figure 1: The Truth table for a logical conditional statement [2] & [3] for my proof in Riemann Hypothesis statement

In reality, a truth table may have some applications in both language or engineering or one may refer to another story. Then, the Riemann Hypothesis statement is said to be both true (for the infinite many examples and compatible with GÖdel's Incompleteness Theorem) or the positive proof and false (for the disproof by the counter-example and incompatible with the GÖdel's Incompleteness Theorem). Thus, in such a case, the Riemann Hypothesis is forced to be true, otherwise either the disproof by the counter-example or the GÖdel's Incompleteness Theorem may be false which cannot happen. All of the above results (true, false cases and the incompleteness) imply and force the Riemann Hypothesis statement must be true or correct. In practice, this author wants to remark that his University of Hong Kong's undergraduate project was once about the discussion in searching the foundation of mathematics by the philosophy (i.e. mathematical logic-ism, intuitionism, formalism and the Gödel's Incompleteness theorem etc where the aforementioned (intuitive) logical truth table (Figure 1) for the Riemann Hypothesis may have some relationship with the previous project's research) from 1995 to 1996 before the Hong Kong handover. Indeed, my UG project's topic was somehow implying a linking with the oil risk in 1970s and the associated commercial economic threats etc.

Literature Review -- A Proof for the infinite many of Riemann Non-Trivial Zeros

Proof for ininite number of Primes

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We may show that there are infinite many of prime numbers with the algebraic philosophical proofs as outlined in [8] & [9]. Then we may apply the prime and non-trivial Zeros fourier duality [10] & [12] to prove that there are actually infinite many Riemann non-trivial zeta zeros. In practice, this author suggests one may find the alternative proof for infinite many number of primes by point set topology as outlined in [11], [12], [13], [14] & [15]. This author will leave the actual proof to those interested parties. However, no matter what kind of proof(s) that you may prefer, the key focus is to find out the contradiction(s) behind the initial assumption(s) and the final result(s) that obtained etc.

Proof for Infinite number of non-trivial zeta zeros

The basic idea of the proof for infinite number of non-trivial zeta zeros by the analytic number theory is: we may first assume that there were finite number of non-trivial zeta zeros. Then from the Riemann Explicit formula [16]:

$\psi(x) = \sum_{p^k \le x} lnp = x - \sum_\rho \frac{x^\rho}{\rho} - ln(2\pi) - \frac{1}{2} ln(1 - x^{-2})$

where ρ is the complex Riemann Zeta zeros,

$\frac{d}{dx}\psi(x) = 1 - \sum_\rho x^{(\rho-1)} - \frac{(x^{-3})}{(1-x^{-2})} = 1 - \sum_\rho x^{(\rho-1)} - \frac{1}{x(x^2-1)}$

But as there were finite number of non-trivial zeta zeros, thus $\sum_\rho x^{(\rho-1)}$ = k where k is a complex valued constant, then ψ'(x) = 1 - (a+bi) - $\frac{1}{x(x^2-1)}$ where k' = (1-a) + bi

But the integration [17] of $\frac{1}{x(x^2-1)}$ = - ln |x| + ½ ln |x-1| + ½ ln |x+1| + c or

ψ (x) = - ln |x| + ½ ln |x-1| + ½ ln |x+1| + [(1-a)+bi]x + c ------ (computed ψ(x) formula)
$.\neq.\sum_{p^k \le x} lnp = \sum_{n \le x}$./\(n)

As the fact that

$\psi(x) = \sum_{p^k \le x} lnp = x - \sum_\rho \frac{x^\rho}{\rho} - ln(2\pi) - \frac{1}{2} ln(1 - x^{-2})$ -------- (original ψ(x) formula).

Actually, $\sum_\rho \frac{x^\rho}{\rho}$ is a polynomial with the highest degree of sup{ρ | ρ is a zeta zeros in the control strip}. But we are now only to employ a linear function of order one (i.e. [(1-a)+bi]x to approximate x - $\sum_\rho \frac{x^\rho}{\rho}$ or x - $\sum_\rho \frac{e^{\rho ln(x)}}{\rho}$. As $e^{\rho ln(x)}$ is an index-complex valued exponential function which may seem to make no sense if there were finite number of non-trivial zeros that may imply a linear approximation [(1-a)+bi]x to $e^{\rho ln(x)}$. Actually, the Taylor Expansion for $e^{\rho ln(x)}$ at point "a" for the first 3 terms [40], according to the U.S.A. Mathematic-a (Home Version, 2023) is:

$$e^{ln(a)^\rho} + e^{ln(a)^\rho}\rho ln(a)^{-1+\rho} ln'(a)(x-a) + \left(\frac{1}{2}e^{ln(a)^\rho}\rho ln(a)^{-2+\rho}\right)$$

$$\left(-ln'(a)^2 + \rho ln'(a)^2 + \rho ln(a)^\rho ln'(a)^2 + ln(a)ln''(a)\right)(x-a)^2 + \ldots + o(x-a)^4$$

which is a polynomial of order at least 3 for the summation to all of the complex valued zeta zeros.

So $\sum_\rho \frac{x^\rho}{\rho} = \sum_\rho \frac{e^{\rho ln(x)}}{\rho}$ where ρ = u + viand belongs to those complex valued zeta zeros
={ $\sum_{re^{i\theta} \in \rho}$. $[e^{ln(a)^\rho} + e^{ln(a)^\rho}\rho ln(a)^{-1+\rho} ln'(a)(x-a) + \left(\frac{1}{2}e^{ln(a)^\rho}\rho ln(a)^{-2+\rho}\right)$
$\left(-ln'(a)^2 + \rho ln'(a)^2 + \rho ln(a)^\rho ln'(a)^2 + ln(a)ln''(a)\right)(x-a)^2 + \ldots + o(x-a)^4$
]/ρ}

In reality, the computed ψ(x) is also different from the original ψ(x) by a constant term ln(2π) = $\frac{\xi'(0)}{\xi(0)}$ where x = 0 or 2nπ for n = 1,2… but c in the computed ψ(x) formula may be equal to $\frac{\xi'(x)}{\xi(x)}$ where $0 <x.< 2n\pi$. Thus, a contradiction is occured mainly due to the initial assumption that there were just a finite number of non-trivial Riemann Zeta zeros. Hence, we may conclude that there are infinite number of Riemann non-trivial zeta zeros.

Alternatively, we have: $x(x^2-1)\psi'(x) = x(x^2-1) - (x^2-1)k - 1$
After simplification, $(x^2-1)[x(\psi'(x)-1)+k] = -1$
$[x(\psi'(x)-1)+k] = -1$ or $[x(\psi'(x)-1)+k] = 1$

x = 0 or x = $2^{1/2}$ or x = - ($2^{1/2}$) and $\psi'(x) = \frac{-k-1}{x} + 1$ or $\psi'(x) = \frac{-k+1}{x} + 1$
$\psi(x) = -(k+1) \ln(x) + x + c$ or $\psi(x) = -(k-1) \ln(x) + x + c$
$. \neq . \sum_{p^k \leq x} lnp = \sum_{n \leq x} \Lambda(n)$
$= x - \sum_{\rho} \frac{x^{\rho}}{\rho} - ln(2\pi) - \frac{1}{2} ln(1 - x^{-2})$

which is a contradiction due to the initial assumption that there were finite number of non-trivial zeta zeros. Hence, we conclude that there are infinite number of non-trivial zeta zeros.

A Conversion between the primes and zeta zeros

In fact, we may still need to compute the recurrence formula for those primes by the Golomb's formula. Once if my proposed proof of Riemann Hypothesis is verified or found to be true, then we may compute the corresponding nth term of the Riemann Zeta zeros as shown in [18] & [19]. Theoretically, it is possible that we may compute the (mirror image inverse) n+1th term of the prime from the nth term of the Riemann Zeta zeros from a corresponding recurrence formula. However, the focus of my series of proof is to determine whether the Riemann Hypothesis is true or false. Thus, this author believes that these pair of the recurrence formulas for both primes and non-trivial zeta zeros are in fact out of the scope of the present research paper.

The Extended Proof to Riemann Hypothesis

In order to have the extended proof version to the Riemann Hypothesis, let us first have a transformation of the formal Riemann Hypothesis statement into the conditional one. That says [41],

"All (nontrivial) zeros of all Dedekind zeta functions lie on the so called "critical line"." Or in terms of the conditional statement:
"If the aforementioned statement is true, then all of the Riemann zeros lie on the critical line and the RH must be true."
In other words, "if it is a non-trivial Riemann Zeta Zeros, then they must not lie outside the critical line (or they must lie on the critical line)." However, the mirror converse of the above statement may NOT always be true since it is well known that "NOT all (or every) points that lie on the critical line must be a non-trivial Riemann Zeta zeros."

Then according to p.7 of [2], it only needs to analyise the conditional statement that (the 'if – then' combination in ordinary speech as mentioned below:)

" If Chan was here, then the knife is not his."
Or (Chan was here) → (The knife is not his)

Thus, my reformulated Riemann Hypothesis conditional statement has a similar conditional structure to the above 'if – then' conditional statement.
Or (non-trivial Riemnann Zeta zeros) → (these zeros must not lie outside the critical line)

Therefore, we may establish a similar (or even the same) truth table as shown in the figure 1 of the introduction section as we have in fact verified those situations (in form of truth table) for the analyising my conditional statement in Riemann Hypothesis. To be precise, we may have the same truth table for my conditional statement of RH just as in figure 1. The only left is just to prove different cases in the truth table for another kind of vertification from my previous paper series in the Riemann Hypothesis. The following four cases and the associated three sub-cases in [1] are indeed the details:

Case I: the (assumption)truth for the Riemann Hypothesis statement gives a positive true result and hence implying RH is correct (i.e. true & true imply true – row one of Figure 1). In other words, we want to show for all non-trivial Riemann Zeta zeros, they must not lie outside the critical the critical line. The proof for the Riemann Hypothesis is said to be true has been shown as in my previous paper [4] by employing Matlab programming code for the verification all over the complex infinity plane except the line x = 1 which is a singularity and hence has an infinite many solutions or infinite many cases;

Case II : the (assumption)false for the Riemann Hypothesis statement gives a positive true result and hence implying RH is correct (i.e. false & true imply true – row two of Figure 1). In other words, we want to show that for all normal complex values (i.e. not non-trivial zeta zeros), they lie outside the critical line. The mirror image inverse proof for the above Riemann Hypothesis is said to be true that has been shown in my previous paper [1]'s case III or the sub-case iii. In fact, the wrong assumptions in the Riemann equations will give positive outcome that non-trivial complex valued model equation part equal to $4*\cot(\ln(x))/(x+1)^2$ which is consistent with the result obtained in [1] while real parts equal to 0.5 (which is also shown in [1]) is just the zeros lie on the critical line. In reality, the above non-trivial zeta root model equation is true all over the critical strip up to infinity as in the process of computation [4], the assumption for the Riemann Zeta function (I.e. $\sum_{n=1}^{\infty} \frac{1}{n^s}$) is a summation from one to infinity, so as the calculated zeta root model equation should be validated all over the infinite real-complex plane (i.e. $\sum_{k=1}^{\infty} \frac{1}{e^{(u+vi)ln(k)}} - \frac{(u+vi)(x-k)}{ke^{(u+vi)ln(k)}} + \frac{\left(-\frac{u^2+2uvi+vi^2-u-vi}{2k^2}+\frac{(u+vi)^2}{k^2}\right)(x-k)^2}{e^{(u+vi)ln(k)}}$). In addition, we solve for the prescribed zeta root model equation to get the final zeta model equation answer 0.5 +/- $\{4*\cot[\ln(x)] / (x+1)^2\}i$ for all x over the infinite real-complex plane together with infinite many solutions. But from [6], this author have found that not all of the complex-valued points that lie on the critical line or {0.5 +/- vi / 0.5 +/- {4*cot [ln(x)] / $(x+1)^2$i, v belongs to real} is the non-trivial zeta zeros. Indeed, the non-trivial zeta roots' model equation is only 0.5 +/- {4*cot [ln(x)] / $(x+1)^2$}i with (infinite many) of them lie on the critical line x = 0.5. The above result implies that there are still some complex-valued numbers but NOT zeta zeros that lie on the critical line x = 0.5. Or in the normal statement (relative to the aforementioned mirror image inverse), there are some complex-valued numbers but NOT non-trivial zeta zeros, they lie outside the critical line x = 0.5. Hence, the prescribed RH statement in my present case II is proved. Actually, all of the normal complex values are independent of any critical lines no matter x = 0.5 or x = 1 and they are distributed all over the complex plane.

Case III: the (assumption) true for the Riemann Hypothesis statement gives a (negative) false result and hence implying RH is incorrect (i.e. true & false imply false – row three of Figure 1). In other words, we want to assume that there exists some non-trivial Riemann Zeta zeros with the properties that these zeros may lie outside the critical line. The disproof to the above statement will be shown as:

First assume that 0.5 +/- [4*cot(ln(x)) / $(x+1)^2$]i is just the model equation for the Riemann Zeta function (i.e. $\sum_{n=1}^{\infty}\frac{1}{n^s}$). Then according to the Dirichlet-Eta function [5]: η(s) = $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ where s = u+vi for all values on the complex plane, by applying the Taylor approximation to η(s) with Maple soft and solve it, we may get:

$$h := \frac{e^{(a-1)\pi I}}{e^{(u+vi)\ln(a)}} + \frac{\left(e^{(a-1)\pi I}\pi I - \frac{e^{(a-1)\pi I}(u+vi)}{a}\right)(x-a)}{e^{(u+vi)\ln(a)}} + \frac{\left(-\frac{e^{(a-1)\pi I}\pi^2}{2} - \frac{e^{(a-1)\pi I}(u^2+2uvi+vi^2-u-vi)}{2a^2} - \frac{e^{(a-1)\pi I}(\pi a I-u-vi)(u+vi)}{a^2}\right)(x-a)^2}{e^{(u+vi)\ln(a)}}$$

where its roots are:

$$\left\{a=a, u=\frac{I\pi a^2 - I\pi a x - avi + vix - \frac{3a}{2} + \frac{x}{2} + \frac{\sqrt{-4I\pi a^3 + 8I\pi a^2 x - 4I\pi a x^2 + a^2 - 6xa + x^2}}{2}}{-x+a}, vi=vi, x=x\right\}, \left\{a=a, u = \frac{I\pi a^2 - I\pi a x - avi + vix - \frac{3a}{2} + \frac{x}{2} - \frac{\sqrt{-4I\pi a^3 + 8I\pi a^2 x - 4I\pi a x^2 + a^2 - 6xa + x^2}}{2}}{-x+a}, vi=vi, x=x\right\}$$

which is obviously different from the previous roots found in [1] for $\sum_{n=1}^{\infty}\frac{1}{n^s}$. Or

$$f := \frac{1}{e^{(u+vi)\ln(k)}} - \frac{(u+vi)(x-k)}{k e^{(u+vi)\ln(k)}} + \frac{\left(-\frac{u^2+2uvi+vi^2-u-vi}{2k^2} + \frac{(u+vi)^2}{k^2}\right)(x-k)^2}{e^{(u+vi)\ln(k)}}$$

where its roots are:

$$\left\{k=k, u=\frac{-vik+vix-\frac{3k}{2}+\frac{x}{2}+\frac{\sqrt{k^2-6xk+x^2}}{2}}{-x+k}, vi=vi, x=x\right\},$$
$$\left\{k=k, u=\frac{-vik+vix-\frac{3k}{2}+\frac{x}{2}-\frac{\sqrt{k^2-6xk+x^2}}{2}}{-x+k}, vi=vi, x=x\right\}$$

In fact, both of the roots for $\sum_{n=1}^{\infty}\frac{1}{n^s}$ and $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ are different. Actually, there is a nπ phase difference/shift between their roots as:

Roots model equation for $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ is 0.5 ±.{4*[cos(±.nπ)]cot [±.ln(x)] / $(x+1)^2$}i but roots model equation for $\sum_{n=1}^{\infty}\frac{1}{n^s}$ is 0.5 ±.{4*cot [±.ln(x)] / $(x+1)^2$}i;

If we apply the Taylor approximation for cot(x) = (1/x) – (x/3) & ln(x) = 2(x-1)/(x+1) & takes the limit x tends to infinity, then we may have:

{4*[cos (±.nπ)]*cot [±.ln(x)] / $(x+1)^2$}i tends to 4*{$\frac{-1}{2}$[$\frac{2-n^2\pi^2}{2}$]}/$(x+1)^2$i and {4*cot [±.ln(x)] / $(x+1)^2$}i tends to (2/3)$[1/(x+1)]^2$ i

Obviously, $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ = 0 (although there is a phase shift) for s = 0.5+/-[4*cot(±.ln(x)) / $(x+1)^2$]i .

But by considering the mirror image, converse of the above: $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ = 0 for s = 0.5 +/- {4*[cos (±.nπ)]*cot [±. ln(x)] / $(x+1)^2$}i with the fact that $\sum_{n=1}^{\infty}\frac{1}{n^s}$= 0 where the real part of the above s also has the value 0.5.

This author have shown in [1] that s = 0.5+/-[4*cot(±.ln(x)) / $(x+1)^2$]i is the computed roots model equation for $\sum_{n=1}^{\infty}\frac{1}{n^s}$with the real part only equal to 0.5. But the roots of $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ and $\sum_{n=1}^{\infty}\frac{1}{n^s}$ both have real parts equal to 0.5 which do NOT lie outside the critical line and is contradicting to the assumption that there are some non-trivial zeta zeros lying outside the critical line. The result may thus imply that the statement assumption that there are some non-trivial zeta zeros lie outside the critical line is incorrect. This is just the disproof to my RH statement in the present Case III by the counter-example (only one is enough but we may still have infinite many counter-examples as the Riemann Zeta root model equation always expands all over the real-complex plane with infinite many solutions;

(N.B.1. In the sense of quantum mechanics [31], there may be a scttering by a Square Well. As shown in the exercise 1 of p.312 in [31], the factor $e^{2i\delta}$=$\frac{cot\pm\frac{n\pi}{2}+i}{cot\pm\frac{n\pi}{2}-i}$ in the similar scattered wave [31], the time delay should be: [time delay] = $2\hbar\frac{d\delta}{dE}=\frac{2m}{\hbar k}\frac{d\delta}{dk}$.

But cot (nπ/2) tends to zero, thus in particular, we may get $e^{i\pi}$ + 1 = 0 or a beautiful Euler God's identity equation [33] & [34].

Indeed, the infinite square potential well may be acted as a model to describe the behaviour of an electron that is confined to move within a one dimensional box of finite width or a square arrary of lens to trick the particle (as a quantum bit for each individual len of the array of lens) [32] for a quantum system to compute. Moreover, there will be a Quantum Fourier Transform $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)e^{-ikx}\,dx$, p.27 [31]. Indeed the transform may be in turn acted as the quantum bits or even the quantum circuits whenever the conditions are available. Hence, we may establish our commercial used quantum computer just like our present desktop one. But the present focus of this paper is a supplementary to the proof of Riemann Hypothesis and the details of the quantum computing will be left for those interested parties).

(N.B. 2. For the Dirichlet-Eta function [42], there may be cases that η(s) = $\sum_{n=1}^{\infty}\frac{-1^{(n-1)}}{n^s}$ =0 for s with real part equal to 1 where the non-trivial roots lie outside the critical line x = 0.5. But the fact is that we have the relationship: η(s) = ξ(s) $(1 - 2^{1-s})$. Hence, for all of the Riemann zeta roots with real part equal to 0.5 is also the roots of η(s). So we conclude there are both types of non-trivial zeros lie on the critical line and lie outside the critical line for the Dirichlet-Eta function while a singularity occurs in the line x = 1 for the Riemann Zeta function ξ(s). In reality, ξ(s) may be independent of the Dirichlet-Eta function's another critical line on x =1 while both of the functions are a type of polylogarithm function [43] & [46] which is an equivalent to Hurwitz function [44] or just a special case of Lerch transcendent [45]. The polylogarithm function may be reduced to the natural log function for some special values of s = 1. That may be why my proof for the Riemann Hypothesis by logarithm inequalities should be correct or acting as a key to open the RH problem door and may be reality solve it in [1]. In reality, logarithm is just the inverse of polylogarithm and that is why my trial for the proof in RH is indeed a success. Actually, we may define the polylogarithm as a power series [45] & [46] like:

Lis(z) = $\sum_{n=1}^{\infty}\frac{z^n}{n^s}$

Moreover, the logarithm function has a high connection with entropy and the (quantum) information (theory). In fact, polylogarithm may be applied in the perturbative quantum field theorem [47] which may be highly related to the correlations between the natural forces with quantum mechanics [48]. Actually, the relationship between quantum field theory and the quantum mechanics is something like the "I am in you and you in me" by the British poet, William Blake.)

Case IV: the (assumption) false for the Riemann Hypothesis statement gives a (negative) false result and hence implying RH is true (i.e. false & false imply true – row four of Figure 1). In other words, we want to show that for all of the normal complex values, they must lie outside the critical line. The proof has been shown in my previous paper [1]'s case I & II or the sub-case i & ii. Or the wrong Riemann equation assumption may give the contradictory results for ξ(s) where s = 1 or {s = u + vi where u, v are real numbers / 0.5+/-4*cot(+/-ln(x))i/$(x+1)^2$} in the prime gap difference. In fact, the wrong assumption is just the number theory equations of (*') in [1] but NOT for the line x = 1 with infinite many solutions in the case of ξ(1) over the line x = 1. Indeed, the the prime gap (difference) answers in [1] are always with infinite many solutions NO matter what the initial assumption may be for all j lies between 1 and infinity over the line x = 1. Moreover, for the case {u + vi / 0.5+/-[4*cot(ln(x)) / $(x+1)^2$]i | u & v are real numbers}, it includes the set of all the infinite real-complex plane. Hence, the sub-case ii will have infinite many solutions as shown in the final prime gap (difference) answer in [1] NO matter what the initial assumption may be for all j lies between 1 and infinity within the infinite real-complex plane.

Conclusion

In a nutshell, this author have already proved the statement of Rienamm Hypothesis is true mainly by the multiplicative telescopic method together with the differences in prime gaps [1] and the logical truth table (Figure 1). As shown in the previous proof section, there are infinite many cases or examples (I, II, IV) for the Riemann Hypothesis is said to be true but we still cannot conclude that RH is true because of the GÖdel's Incompleteness Theorem. Thus, we may just say that these truth examples (case I, II, IV) of RH are only compatible true examples [27] with the GÖdel's Incompleteness Theorem. However, there is only one case (III) for the Riemann Hypothesis to be false or RH is disproved by the counter-example [26] in case III. Then in such a case, RH is said to be false or it is the disproof of the Riemann Hypthesis (by a counter-example) which is NOT compatible with the Gödel's undecidability. Otherwise, either my disproof to RH in case III or the GÖdel's Incompleteness Theorem will be false which are both impossible [28]. Therefore, only the infinite many true examples of Riemann Hypothesis must be correct and compatible with the Gödel's undecidability. Or in such a case, the Riemann Hypothesis is thus forced to be correct [29 a & b]. Therefore, we come to a conclusion that this author have proved that the Riemann Hypothesis statement is correct or true. A last word for this author's final remark is that the Riemann Hypothesis statement may be independent of the present ZFC system as one may need to reconstruct a new real number line where x = 0.5 becomes x = 0 in order to contain all non-trivial zeta zeros. Once if we may restore the traditional real number line without a transformation of x = 0.5 to x = 0, then the Riemann Hypothesis statement may depend on the old ZFC system. Hence, we may need to employ a (hybrid) fuzzy between the independence and dependence or a "(Hybrid) Fuzzy ZFC system" [30] for solving the Riemann Hypothesis. In practice, no matter the dependence or the independence, the key interests will be to further investigate both the structure and the random-ness of those non-trivial zeta zeros together with an application in the (quantum) cryptography etc.

Limitations -- An Error Estimate for the Computed Root's Model Equation

In practice, the computed Riemann Zeta function (i.e.$\sum_{\square=1}^{\infty}\frac{1}{\square^{\square}}$)'s root model equation is 0.5+/-[4*cot(ln(x)) / $(x+1)^2$]i. It tends to (2/3)$[1/(x+1)]^2$ when x tends to infinity for cot(ln(x)) where cot(x) = (1/x) – (x/3) & ln(x) = 2(x-1)/(x+1).

Actual Zeta Root	Estimated Zeta Root	Absolute Percentage Error
14.1347	14 -- (for (2/3)$[1/(x+1)]^2$ = [14.1347])	0.95297%
21.0220	21 -- (for (2/3)$[1/(x+1)]^2$ = [21.0220])	1.04%
25.0109	25 -- (for (2/3)$[1/(x+1)]^2$ = [25.0109])	0.0436%
30.4249	30 (for (2/3)$[1/(x+1)]^2$ = [30.429])	1.40984%
32.9351	33 (for (2/3)$[1/(x+1)]^2$ = [32.9351])	0.1971%
37.5862	38 (for (2/3)$[1/(x+1)]^2$ = [37.5862])	1.1009%
40.9187	41 (for (2/3)$[1/(x+1)]^2$ = [40.9187])	0.1987%
43.3271	43 (for (2/3)$[1/(x+1)]^2$ = [43.3271])	0.75495%
48.0052	48 (for (2/3)$[1/(x+1)]^2$ = [48.0052])	0.01083%
49.7738	50 (for (2/3)$[1/(x+1)]^2$ = [49.7738])	0.45446%

Figure 2. Absolute Percentage Error between the first to the tenth estimated (by s = 0.5+/-[4*cot(ln(x)) / $(x+1)^2$]i) & the actual Riemann zeta root

It seems that the maximum absolute percentage error is about 1.5% for the first ten Riemann Zeta zeros which lies in an acceptable range. This also constitues another positive result for the disproof for Riemann Hypothesis by the counter-example. In fact details for the structure of the Riemann Zeta zeros should be investigated by the complex Lie Algebra and Lie Groups [21] together with the corresponding Branching rule(s) [20] & [23] discovered by the mathematic-a software LieART 2.0 [22] or even the complex Lie Algebra as well as the homology & homotopy etc [24]. In fact, by considering the lattice's weight unification and decomposition so as to understand the complexity of the processing system [25]. Actually, the determination of the structure for the Riemann Zeta zeros is another story for researching which is out of the scope of the present topic in the dtermination whether the statement of Riemann Hypothesis is true or false. Finally, we may observe that those non-trivial Riemann Zeta Zeros can be truncated [37] & [38] and hence is measurable or we may "quantum mechanics" those Zeta Zeros [35] & [36].

REFERENCES

1. Lam Kai Shun (2023) A Full and Detailed Proof for the Riemann Hypothesis & the Simple Inductive proof of Goldbach's Conjecture, *International Journal of Mathematics and Statistics Studies,* Vol.11, No.3, pp.1-10
2. K.T. Leung & Doris L.C. Chen, (1964) Elementary Set Theory, Hong Kong University Press.
3. Hanna, G., & de Villiers, M. (Eds.) (2012). Proof and proving in mathematics education: The 19th ICMI study. Springer. https://doi.org/10.1007/978-94-007-2129-6
4. Lam Kai Shun (2023) A Verification of Riemann Non-Trivial Zeros by Complex Analysis by Matlab™ Computation, European Journal of Statistics and Probability, *European Journal of Statistics and Probability,* 11 (1) 69-83.
5. Wolfram MathWorld, Dirichlet-eta Function https://mathworld.wolfram.com/DirichletEtaFunction.html
6. Shun L.K. (2023) The Quantized Constants with Remmen's Scattering Amplitude to Explain Riemann Zeta Zeros, *International Journal of English Language Teaching,* Vol.11, No.4, pp.,20-33
7. D.L.Livingston, 1985. The Unification and decomposition of processing structures using lattice theoretic methods, Old Dominion University, https://digitalcommons.odu.edu/cgi/viewcontent.cgi?article=1182&context=ece_etds
8. Dr. Trefor Bazzett, Proof: There are infinitely many prime numbers, https://www.youtube.com/watch?v=inUkhh8-h-I
9. Jean-Marie De Koninck & Florian Luca, 2010. Analytic number theory: exploring the anatomy of integers, *Americian Mathematics Society,* p.21, Section 2.2.
10. University of Exeter, the "encoding" of the distribution of prime numbers by the nontrivial zeros of the Riemann zeta function [common approach] https://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/encoding1.htm
11. H.Furstenberg, 2024. A topological proof of the infinite of primes. https://www.youtube.com/watch?v=lpS7uuyZLtk
12. University of Exeter, Fourier analysis and number theory, https://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/NTfourier.htm
13. MetaMaths, Infinitude of Primes using Topology. Furstenberg's proof https://www.youtube.com/watch?v=mscgSvJfNzA

14. Math Geeks, Furstenberg’s proof of the infinitude of primes (topological proof) https://www.youtube.com/watch?v=yVRafEDBy5E
15. Furstenberg, Harry(1955). "On the infinitude of primes". *American Mathematical Monthly*. 62 (5): 353. doi:10.2307/2307043. JSTOR2307043. MR0068566.
16. D.Harold, 1967. Multiplicative Number Theory, 3rd Edition, Springer Publication
17. Socratic, 2017. https://socratic.org/questions/how-do-you-evaluate-the-integral-int-1-x-3-x-dx
18. Kawalec. A., 2020 Analytic recurrence formulas for non-trivial zeros of the Riemann Zeta function, https://arxiv.org/abs/2012.06581
19. Kawalec, A., 2021 The recurrence formulas for primes and non-trivial zeros of the Riemann Zeta function, https://arxiv.org/pdf/2009.02640.pdf
20. Lingxiao Li, (2018) Branching Rules of Classical Lie Group in Two Ways, Undergraduate Honors Thesis, Department of Mathematics, Stanford University.
21. W. Fulton., J. Harris, (2004) Representation Theory – A First Course , Springer Publication, Chapter 21 & 23
22. Hanna. G, Michael de Villiers, (2011) Proof and Proving in Mathematics Education, The 19th ICMI Study, *International Commission on Mathematical Instruction,* Springer Publication.
23. R.J.Saskowski, T.W.Kephart (2019) LieART 2.0 – An Improved Way to Compute Branching Rules, Undergraduate Honors Thesis in Physics, Department of Physics & Astronomy Vanderbilt University, Nashville.
24. W.Fulton, 1995. Algebraic Topology – A First Course, Graduate Texts in Mathematics, Springer Publication.
25. D.L.Livingston, 1985. The Unification and decomposition of processing structures using lattice theoretic methods, Old Dominion University, https://digitalcommons.odu.edu/cgi/viewcontent.cgi?article=1182&context=ece_etds
26. Any counter example for this claim?, Mathematics Stack Exchange, https://math.stackexchange.com/questions/1628063/any-counter-example-for-this-claim?rq=1
27. Understanding Godel’s Incompleteness Theorem, Mathematics Stack Exchange, https://math.stackexchange.com/questions/16358/understanding-g%C3%B6dels-incompleteness-theorem
28. Godel’s Incompleteness Theorem: How can truth go deeper than proof?, Philosophy Stack Exchange, https://philosophy.stackexchange.com/questions/90302/g%C3%B6del-s-incompleteness-theorem-how-can-truth-go-deeper-than-proof
29. [a] Dawson, John W. Jr. (1996). Logical dilemmas: The life and work of Kurt Gödel. Taylor & Francis. ISBN.
[b] Dawson, John W. Jr. (1997). Logical dilemmas: The life and work of Kurt Gödel. Wellesley, Massachusetts: A. K. Peters. ISBN. OCLC36104240
30. Paulson, Lawrence(2014). "A machine-assisted proof of Gödel's incompleteness theorems for the theory of hereditarily finite sets". Review of Symbolic Logic. 7 (3): 484–498. arXiv:2104.14260. doi:10.1017/S1755020314000112. S2CID13913592
31. H.C. Ohanian (1990), Rensselaer Polytechnic Institute, Principles of Quantum Mechanics, Prentice Hall/Englewood Cliffs, New Jersey 07632, U.S.A.
32. Bluvstein, D., Evered, S.J., Geim, A.A. et al. Logical quantum processor based on reconfigurable atom arrays. *Nature, 626*, 58–65 (2024). https://doi.org/10.1038/s41586-023-06927-3
33. Gallagher, James (13 February 2014). "Mathematics: Why the brain sees maths as beauty". BBC News Online. Retrieved 26 December 2017
34. Wilson, Robin (2018). Euler's pioneering equation: the most beautiful theorem in mathematics. Oxford: Oxford University Press. ISBN
35. Quantum Mechanics – the nontrivial Riemann Zeta Zeros interpreted as a spectrum of energy levels, https://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/physics1.htm
36. He, R., Ai, MZ., Cui, JM. et al. Riemann zeros from Floquet engineering a trapped-ion qubit. npj Quantum Inf 7, 109 (2021). https://doi.org/10.1038/s41534-021-00446-7
37. Tao, T. (2011). An introduction to measure theory. American Mathematical Society.
38. Chan, J. (1987). Compact convex sets and their affine function spaces. (Thesis). University of Hong Kong, Pokfulam, Hong Kong SAR. Retrieved from http://dx.doi.org/10.5353/th_b3042584
39. Apostol T.M. (1974), Mathematical Analysis, Addison Wesley, Second Edition, U.S.A.
40. Sagan, B.E. Combinatorics: The Art of Counting, American Mathematical Society
41. Exeter University, Riemann Hypothesis, https://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/riemannhyp.htm
42. Kucs, Tibor, Dirichlet Eta Zeros on the Line Re(s) = 1 (January 17, 2022). Available at SSRN: https://ssrn.com/abstract=4011285or http://dx.doi.org/10.2139/ssrn.4011285
43. Apostol, T.M. (2010), "Polylogarithm", in Olver, Frank W. J.; Lozier, Daniel M.; Boisvert, Ronald F.; Clark, Charles W. (eds.), NIST Handbook of Mathematical Functions, Cambridge University Press, ISBN , MR 2723248
44. Berndt, Bruce C. (Winter 1972). "On the Hurwitz zeta-function". *Rocky Mountain Journal of Mathematics.* 2 (1): 151–158. doi:10.1216/RMJ-1972-2-1-151. Zbl 0229.10023
45. Garunkstis, Ramunas (2004). "Approximation of the Lerch Zeta Function" (PDF). *Lithuanian Mathematical Journal.* 44 (2): 140–144. doi:10.1023/B:LIMA.0000033779.41365.a5. S2CID 123059665
46. Edwards, H.M. (1974) Riemann’s Zeta Function. Dover Publications, Inc., New York.
47. Pennington, Jeffrey S. 2013, Generalized polylogarithms in perturbative quantum field theory, Phd thesis, Stanford University, https://inspirehep.net/literature/1449941
48. Kai Shun, CL., 2020, Can Quantum Mechanics Correlate All Natural Forces? An Experimental and Observational Approach, Vol.11. 3, *Journal of Physical Mathematics,* DOI: 10.37421/jpm.2020.11.318

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