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Research Article

ARCHITECTURAL DESIGN RULES OF SOLAR SYSTEM BASED ON KINEMATIC MODEL PREDICTS AN EARLY DEMISE OF PHOBOS - A MOON OF MARS

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Abstract

Mars-Phobos-Deimos is a text book example of Architectural Design Rules of Solar Systems derived from Kinematic Model (KM) of tidally interacting binaries. Architectural Design Rules predict that the Phobos and Deimos have been accreted in the equatorial plane of Mars from the extra-martian impact generated debris disk and are born at inner Clarke's orbit (a_{G1}) in kinematic model (KM) also known as synchronous orbit $a_{asyn} \approx 6$ Mars radii (R_{Mars}) in classical mechanics about 4.5Gy ago. Since (a_{G1}) is an energy maxima hence it is an unstable equilibrium orbit and slightest perturbation causes Phobos to tumble short of (a_{G1}) and Deimos to tumble long of (a_{G1}). As a result Phobos is launched on a sub-synchronous orbit rapidly spiraling-in to its certain doom on a gravitationally runaway collapsing spiral path very similar to in-spiral, merger and ring down of black hole binaries. Deimos is launched on a super-synchronous orbit spiraling out at snail's pace because of vanishingly small mass ratio of the moon and planet. Presently Phobos is at $2.9R_{Mars}$ well within Roche's Limit of $3.02R_{Mars}$ and hence it is under considerable tidal stress and may undergo material's failure and may form a ring around Mars much as we have a Saturnian ring. If it survives the tidal stress it will make a direct collision with Mars in 10My from now since it is rapidly losing its altitude at the rate of 21cm/y according to KM. A recent advanced simulation study by Canup and Salmon (2018) confirm all the findings mentioned above except the doomsday of Phobos thereby corroborating the theoretical results of KM and validating KM itself.. Recent Mars Express (Burns 1978, Witasse et.al 2013) results show that the altitude loss is at 1.8cm/yr and the doomsday will occur in 100My. Bills *et al. (*2005) and Ramslay & Head III(2013) have reported altitude loss rate at 4cm/yr and doomsday at 30-50My. Black and Tushar (2015) have predicted that, in 20 to 40My, Phobos will be tidally pulverized into a dust ring around Mars at (~ $1.6R_{Mars}$). The doomsday of Phobos according to Minton and Hesselbrock (2017) is at 70My.All the above results are based on elasto-viscous model of tidally interacting binary. Whereas present research is based on KM. The ultimate validation of these results will come from future Interplanetary Laser Ranging Missions(ILRM) notably from Phobos Laser Ranging Mission(PLRM) .

Keywords: Phobos Laser Ranging Mission, Interplanetary Laser Ranging Missions, Death spiral, Altitudinal decay rate, Synchronous orbits, Inner Clarke's orbit, Outer Clarke's orbit.

INTRODUCTION

The dawn of Lunar Laser Ranging Experiments (20th July 1969)

George Gamow's book 'The Story of Moon' first introduced the Author to the tidally interacting Earth-Moon system in 1960's. It described the tidal brake being applied to rapidly spinning Earth, the gradual slowdown of Earth's spin period from 5 hours to present 24 hours, tidally evolving expanding spiral path of Moon from its birth orbit 18, 000Km from the center of Earth to the present lunar orbit of 384, 400Km and it gave a data set of Length of (Earth) Day in the past geologic epochs. These facts were further verified through Issac Asimov's popular book on Science and Technology and through Carl Sagan's layman's book and TV series COSMOS. The Author endeavored to solve the equation of motion based on the orbital motion of Moon around Earth. It was a second order ordinary linear differential equation and required two boundary conditions for its complete solution. On 20th July 1994, the Silver Jubilee Anniversary of Man's landing on Moon was celebrated. This was the day 25 years earlier when Neil Armstrong had radioed from Moon's surface,

"Houston ! This is Tranquility Base here. Eagle has landed. This may be a small step for Man but a giant leap for Mankind".

In Apollo 11 mission, Michael Collins was in the command module while Neil Armstrong and Aldrin Buzz were in the lunar module named the Eagle. In that mission three experiments were set up namely: retro-reflector for Lunar Laser Ranging experiments, seismograph for the study of Moon's interior and aluminum foil for collecting the cosmic particles and ascertaining Lithium content. On 20th July 1994, NASA issued a historic press release:

"Moon had receded by 1 meter in last 25 years from 20th July 1969 to 20th July 1994".

By Lunar Laser Ranging experiment it is well established that Moon is receding at 3.82±0.07 cm/y (Dickey *et al.* 1994) and this has become a powerful tool for scientific investigations and discoveries. LLR has contributed to the revolutionary breakthrough in the, lunar ephemeris, with a three-orders-of-magnitude improvement in accuracy; a several-orders-of magnitude improvement in the measurement of the variations in the Moon's rotation; and the verification of the principle of equivalence for massive bodies with unprecedented accuracy (Dickey *et al.* 1994).

Development of Kinematic Model of Earth-Moon System

The Author redid E-M calculation and determined the time integral from the birth orbit to the present orbit. Arbitrary constants were adjusted to obtain a transit time of 4.5Gy.A paper was presented in 82nd India Science Congress-1995

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(Sharma 1995). The Author further elaborated the E-M system dynamics and determined the theoretical Length of Day curve and superimposed on the observed curve of LOD. It was predicted that deviation of the real time observed LOD curve with respect to the theoretical LOD curve may contain precursors for impending Earthquakes and sudden volcanic eruptions and this could become a powerful tool for Early Warning and Forecasting Method (EWFM) for earthquakes and sudden volcanic eruptions.. This was presented at World Space Congress 2002 (34th COSPAR Scientific Assembly) held at Houston, USA (Sharma and Ishwar 2002).

E-M system dynamics led the Author to a generalized (Length of Month/Length of Day) or LOM/LOD equation (see supplementary materials Sharma 2023B) namely:

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = A \times a^{3/2} - F \times a^2 \qquad 1$$

Where

$$A = \frac{J_T}{B \times C}; F = \frac{m}{1 + \frac{m}{M}} \times \frac{1}{C}; B = \sqrt{G(m + M)};$$

$C = principal\ moment\ of\ inertia\ of\ Earth\ around\ its\ spin\ axus,$
$G = gravitational\ constant; M = mass\ of\ Earth; m = mass\ of\ Moon;$
$J_T = total\ angular\ momentum\ of\ E - M\ system;$
$\Omega = E - M\ orbital\ angular\ velocity\ and\ \omega = spin\ angular\ velocity\ of\ Earth.$

In our case Moon is locked with Earth in a synchronous orbit hence :

$$\Omega(orbital\ angular\ velocity) = \omega^*(spin\ angular\ velocity\ of\ Moon)$$

For compact binaries and for regular moons (moons in equatorial plane of the host planet) and its host planet this is always true.

Total angular momentum of E-M system should be the vector sum of spin angular momentum of Earth, orbital angular momentum and spin angular momentum of Moon including the obliquity angle of Earth (φ), inclination angle of Lunar orbital plane (α) and obliquity of Moon's spin axis (β) with respect to Lunar Orbital Plane normal.

For mathematical tractability, the scalar sum was considered and $\varphi = \alpha = \beta = 0$ assumed. This simplified model was called Kinematic Model (KM). Only recently Advanced Kinematic Model (AKM) has been developed where total angular momentum was considered as vector sum and actual values of Earth's obliquity, Lunar orbital plane inclination and Moon's obliquity are included. Two papers based on Advanced Kinematic Model have been published in peer reviewed journals (Sharma 2023C and Sharma 2024B).

In this paper we follow KM.

The Author found that Equation (1) can be set up for any two-body system. Substituting the Globe-Orbit parameters for the given binary system, Equation 1 was found to predict LOM/LOD or ω/Ω for the given binary at the given orbital radius with less than 5% error. This error is within the margin of error in the Globe-Orbit parameters.

Equation (1) when equated to UNITY gave two roots for E-M system which in essence are geo-synchronous orbits for E-M system.

$$a_{G1} = inner\ geo - synchronous\ orbit\ and\ a_{G2} = outer\ geo - synchronous\ orbit\,.$$

In generalized binaries they will be referred to as:

$$a_{G1} = inner\ Clarke's\ orbit\ and\ a_{G2} = outer\ Clarke's\ orbit$$

Geo-synchrony in E-M system implies triple synchrony state which is defined as:

$$\omega(spin\ angular\ velocity\ of\ Earth) = \omega^*(spin\ angular\ velocity\ of\ Moon) = \Omega\ (orbital\ angular\ velocity\ of\ E - M\ system) \qquad 2$$

Triple synchrony state is really tidal interlocking of the primary and Secondary. In this state the long axis of tidally oblate primary and secondary are aligned and the two rotate around the bary-center of the system as one body. Pluto-Charon is a text-book example for triple synchrony and interlocking between primary and secondary (Sharma & Ishwar 2004A).

According to KM conjecture, satellites (or the secondary) are born at a_{G1} either by accretion or by capture but a_{G1} being an energy maxima (see supplementary material Sharma 2024) and hence being an unstable equilibrium orbit the secondary tumbles long or short of a_{G1} at the slightest perturbation by solar wind, cosmic particles or radiation pressure. As shown in supplementary materials Sharma 2024, a_{G1} is the energy maxima hence an unstable equilibrium orbit where as a_{G2} is the energy minima a stable equilibrium orbit.

In E-M system, a_{G1} is ~15, 000Km and Roche's Limit (the orbit within which the particles cannot accrete due to tidal stress and if an intact solid body enters this limit it gets tidally pulverized) a_R is $(16\rho_{Mars}/\rho_{Phobos})^{1/3} \times R_{Mars}$ ~ 18, 000Km (Ida and Stewart 1997). Hence Moon forms by accretion from impact generated circum-terrestrial debris in Roche's zone beyond 18, 000Km and it is unconditionally launched on super-synchronous orbit. Initially tidal dissipation is negligible because it is a triple synchrony state and there is no stretching and squeezing of Earth but Earth spin is being slowed by Moon due to the fact that Earth's tidal bulge leads the E-M radius vector (see supplementary notes Sharma 2023A). As Earth's spin is slowed, both angular momentum and rotational kinetic energy are transferred to Moon's orbit. This creates an impulsive torque which acts like gravitational sling shot for catapulting Moon on an expanding spiral path (Cook 2005, Dukla et al 2004, Epstein 2005, Jones 2005). But very soon conservative phase ends and Earth experiences tidal heating. The extent of tidal heating can be gauged from the volcanic activity in 'Io' a natural satellite of Jupiter. Today Io is the most volcanically active body in our Solar System and this volcanic hyperactivity is a result of Io's eccentric orbit due to 2:1 Mean Motion Resonance with Europa and the consequent tidal heating (Lopes *et al.* 2004). As Moon recedes, the differential in ω and Ω grows and there is considerable stretching and squeezing of Earth which causes tidal heating of Earth. Once Earth enters tidal heating phase no rotational energy is transferred from Earth to Moon but angular momentum continues to be transferred from Earth to Moon and Moon continues to recede on an expanding spiral path (Sharma *et al.* 2009). This recession takes place because Moon continues to coast and climb up the gravitational potential well by virtue of the initial energy acquired by it during the conservative gravitational sling shot phase and by virtue of inertia. Moon continues to traverse an expanding spiral path until billion years later it will lock into the outer geo-synchronous orbit which is at a_{G2} ~ 552, 587.89Km. At this point E-M system enters a triple synchrony state described by (2), all tidal dissipation stops, the orbit is circularized and both Earth and Moon are synchronized at 47 days orbital period=47 days spin period of Earth =47 days spin period of Moon (Darwin 1889, 1890).

In final lock-in position, the long axis of tidally oblate Earth and the long axis of tidally oblate Moon are perfectly aligned and the two components move as one body around the bary-center of E-M system in 47 days. This scenario has been achieved by Pluto-Charon system (Sharma and Ishwar 2004A). E-M final lock-in will continue until Sun gravitational perturbation breaks the triple synchrony of E-M system and Moon is again launched on a sub-synchronous spiral-in path. How does this break-up of the triple synchrony occur ?

In the final triple synchrony state :

Earth spin = 47days, Moon's spin = 47 days and E-M orbital period = 47 days. This had been determined by George Howard Darwin. (Darwin 1879, Darwin 1880). But Earth's orbital period around Sun is 365.25 days. Hence Earth will be de-spun to 365.25 days by Sun's tide on Earth. As Earth spins down, the triple synchrony of E-M system breaks and E-M enters sub-synchronous state and Moon is launched on a collapsing spiral orbit also known as death spiral (Jeans 1936).

In Mars-Phobos-Deimos system, according to KM conjecture Phobos and Deimos. Both are born by accretion from circum-martian impact generated disk(Canup & Salmon 2018, Peale & Canup 2015) at a_{G1} but a_{G1} is energy maxima as discussed in supplementary materials Sharma 2024 hence it is an unstable equilibrium and slightest perturbation causes the two moons to tumble on two sides of a_{G1}. Phobos tumbles short of a_{G1} and Deimos tumbles long of a_{G1}. Deimos is in super-synchronous orbit and by gravitational sling shot effect it is launched on an expanding spiral orbit (Sharma *et al.* 2009) but the mass ratio of Deimos and Mars is negligibly small hence after the gravitational sling shot effect terminates it coasts on a expanding spiral path where it has traversed spirally from $6R_{Mars}$ to 6.9 R_{Mars} in 4.5Gy (Thomas & Veverka 1980) which is snail's pace. Time constant of evolution (Sharma 2011) is inverse power of mass ratio (10^{-8}) and here it is in Gega years therefore it moves at snail's pace. In Mars-Phobos, Phobos is launched on a sub-synchronous orbit which is gravitationally runaway collapsing spiral path or also known as a death spiral. Runaway implies in-built positive feedback loop which propels the process inexorably to an energy minima.. In sub-synchronous orbit, Mars tidal bulge lags the Mars-Phobos radius vector hence rapidly orbiting Phobos (6h) accelerates the Mars angular spin(24h) leading to rotational energy and angular momentum transfer from Phobos to Mars (see supplementary materials Sharma 2023A). It is this transfer which causes the decay of orbital radius leading to further spin up of Mars which leads to further transfer of rotational energy and angular momentum. *This sets up a positive feedback loop which leads to a gravitational runaway collapsing orbit.* Hence though the mass ratio is 10^{-8} and tidal interaction is weak but it is in gravitational runaway phase hence in 4.5Gy its semi-major axis has decayed from $6R_{Mars}$ to 2.9 R_{Mars}. As shown in Figure 4 and Figure 5, this in-spiral bears resemblance to Black Hole binary in-spiral, merger and ring-down, see Figure 6, and produces a similar chirp signal (Mehata *et al.,* 2017).According to KM, altitude is being lost at 21cm/y rate leading to either the formation of a ring at 1.6 R_{Mars} (Black & Mittal 2015) or a catastrophic collision of Phobos with Mars in 10My which is much earlier then what is predicted by other scientists based on seismic considerations (see supplementary materials Sharma 2023A).

From Equation 1, velocity of recession of Moon can be determined.(see supplementary materials Sharma 2023B);

Velocity of Recession/Approach

$$V = \frac{da}{dt} = \frac{K}{a^Q}\left(A \times a^2 - F \times a^{\frac{3}{2}} - \sqrt{a}\right)\left(\frac{2}{m^* B}\right)$$

$$where\ a = semi - major\ axis\ of\ the\ lunar\ orbit;\ m^* = reduced\ mass\ of\ Moon = \frac{m}{1 + m/M};$$

The above equation gives meter per second hence to get a measurable value it must be multiplied by 31.5569088×10^6 second/solar year to get the answer in m/y.

Hence the final equation is:

$$V = \frac{da}{dt} = \frac{2K}{m^* B} \times \frac{1}{a^Q}\left(A \times a^2 - F \times a^{\frac{3}{2}} - \sqrt{a}\right) \times 31.5569088 \times 10^6 \frac{m}{y} \quad 3$$

K is structure constant and Q is exponent. These are to be determined from the boundary conditions.

Equation 3 gives the Velocity of Recession if da/dt is positive and Velocity of Approach if da/dt is negative. For super-synchronous orbits, it is always positive and the secondary is receding. For sub-synchronous it is always negative and the secondary is approaching the primary. Equation 3 can be utilized to determine the transit time from any earlier orbit (a_1) to the present orbit(a_2) from the following time integral:

$$Transit\ Time = NIntegrate\left[\frac{1}{V(a)} da, \{a, a_1, a_2\}\right] \quad 4$$

In February 2004, the Author found that LOM/LOD equation could as well be set up for Sun and Planets and also for exo-solar systems. After examining a large number of Planets - Sun and Exo-planet and Planet Hosting Star the New Perspective on Solar and Exo-solar systems was proposed at 35th COSPAR Scientific Assembly held on 18th to 25th July 2004 at Paris, France (Sharma and Ishwar 2004B). This New Perspective was developed into Architectural Design Rules of Solar Systems at CELMEC V in 2009 at Viterbo, Italy (Sharma 2011).

Architectural Design Rules based on the New Perspective

The Author (Sharma 2023B) presented a paper titled Iapetus hypothetical sub-satellite re-visited and it reveals celestial body formation process in the KM Framework. In that paper the Author derived the dynamical behavior of tidally interacting binaries for a whole range of mass ratios q (secondary mass/primary mass) ranging from negligibly small to Unity. The holistic behavior of the tidally interacting binary is summed up in Figure 1. Here R_{Iap} = volumetric mean radius of Iapetus (the third largest moon of Saturn). Sub-satellite(SS) and Iapetus constitute the tidally interacting binary and a_{synSS} = classical synchronous orbit of SS around Iapetus and a_{G1} and a_{G2} are the inner and outer Clarke's orbit of Iapetus-SS binary system based on KM.

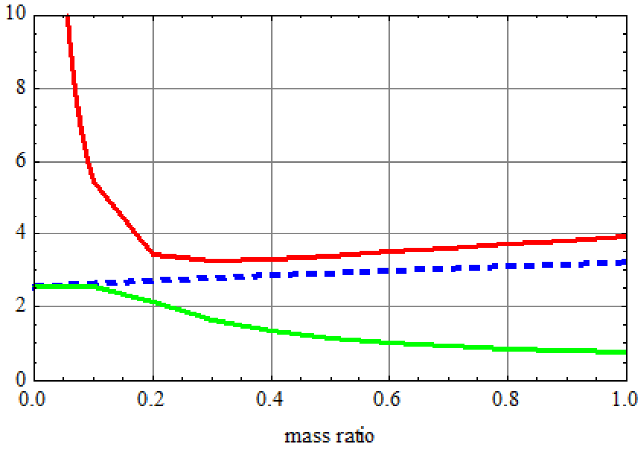

Figure 1. [.tiff].Plot of a_{synSS} (×R_{Iap})[Dashed Blue], a_{G1} (×R_{Iap})[Thick Green] and a_{G2} (×R_{Iap})[Thick Red] as a function of 'q'=mass ratio. Y-axis is semi-major axis as a multiple of Iapetus Globe Radius. [Courtesy:Author]

Inspection of Figure 1, tells us that

Theorem 1: all tidally interacting binaries have two triple synchrony orbits also known as Clarke's orbits in generalized binaries or Geo-synchronous orbits in case of E-M system.

Theorem 2: at infinitesimal values of 'q', a_{synSS} is the same as a_{G1} and only inner Clarke's Orbit is perceptible.

Theorem 3: at larger mass ratios the two (classical synchronous orbit and kinematic formalism for a_{G1}) rapidly diverge. Author's analysis till now has confirmed that a_{G1} is the correct formalism for predicting the inner triple synchrony orbit in a binary system for q < 0.2.

Theorem 4:At mass ratios greater than 0.2, a_{G1} is physically untenable and only a_{G2} is perceptible. [Outer Triple Synchrony Orbit or outer Clarke's orbit seems to converge but does not actually converge to the classical formalism but remains offsetted right till the limit of q =1. Here again only outer Clarke's Orbit is perceptible but the actual Star pairs satisfy the Kinematic formalism and not the classical formalism.]

Theorem 5: For mass ratio less than 0.0001, binaries remain in inner Clarke's Configuration stably which is predicted by Classical Formalism also.

Theorem 6: At mass ratios greater than 0.2 right up to unity, star pairs remain in outer Clarke's Configuration stably and its magnitude is more than classical synchronous prediction.

Theorem 7:For mass ratios 0.0001 < q < 0.2, Outer Clarkes configuration is the only stable orbit and secondary is catapulted from a_{G1} by Gravitational Sling Shot mechanism and it migrates out of that configuration. If it is at a > a_{G1} the pair spirals out with a time constant of evolution (see Sharma 2011) and if a < a_{G1} then the pair spirals-in along a gravitational runaway collapsing spiral path on a collision course with the primary.

Time Constant of Evolution is in inverse proportion of some power law of mass ratio (Sharma 2011).

For q = 0.0001, it is Gy and as q increases, time-constant decreases from Gy to My to kY to years. This is valid for the range of masses encountered in Solar and Exo-Solar Systems. Between 0.2 to 1, a solar nebula falls into outer Clarke's Configuration by hydro-dynamic instability within months/years to form star pairs.[see supplementary notes Sharma 2023B, Table 1].

For q being vanishingly small, the calculation of the man-made Geo-synchronous Satellite's orbit of 36, 000Km above the equator has been done by Kinematic Formalism. This calculation has been done by the Author in his personal communication: http://arXiv.org/abs/0805.0100. The geo-stationary orbit (36, 000Km above the equator) for communication satellites in Communication Text books has been calculated from classical synchronism condition.

Theorem 1 is applicable in Mars-Phobos-Deimos (M-P-D) case. M-P-D is text-book example of Architectural Design Rules of Solar System and the orbital configurations of Phobos and Deimos are illustrated in Figure 2 and Figure 3. Architectural layout parameters are tabulated in Table 1.

Table 1. Architectural lay-out parameters of Phobos and Deimos around Mars

	Phobos	Deimos	Comments
Age	4.5Gy	4.5Gy	
'a'(present)(Km)	9, 830	23, 450	
'a'(present)(×R_E)	2.9	6.9	
'a_R'(Km)Roche's Limit	10, 885.1	11, 187.6	
'a_R'(×R_E) Roche's Limit	3.2	3.3	
a_{G1}(Km), (×R_E)	20, 423; 6.02	20, 423; 6.02	
a_{G2}(Km)	7.4589×10^{15}	1.68998×10^{17}	Beyond perception
Time constant of evolution	Giga years	Giga years	Practically no tidal evolution for Deimos.

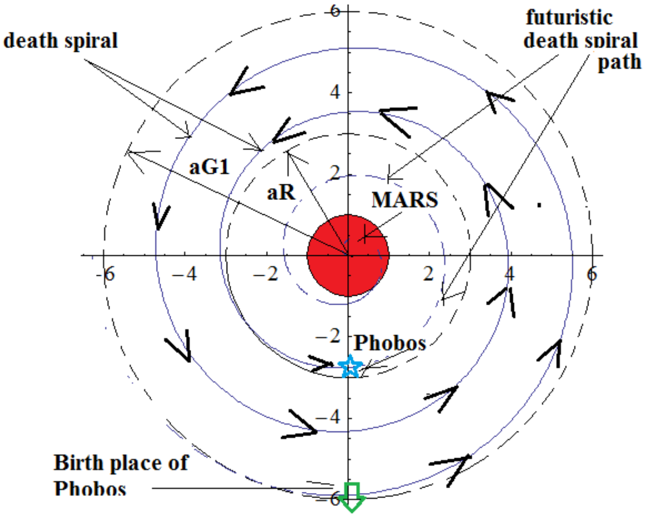

Figure 2. 2-D Orbital configuration of Phobos around Mars trapped in sub-synchronous orbit also known as death spiral. [Courtesy:Author]

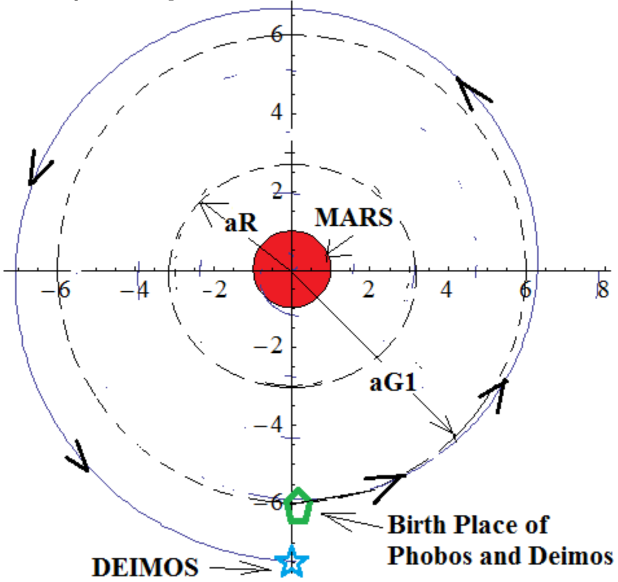

Figure 3.2-D Orbital configuration of Deimos around Mars in a super-synchronous orbit. [Courtesy:Author]

Figure 2, Figure 3 and Table 1 clearly show that both moons Phobos and Deimos originated at inner Clarke's orbit = $6R_{Mars}$ from an impact generated circum - martian accretion disk. Phobos tumbled short of inner Clarke's orbit hence as shown in Figure 2, Phobos is trapped in a death spiral (gravitationally runaway collapsing spiral path). Here tidal interaction is very weak but it is caught in a gravitationally runaway path hence in 4.5Gy it manages to spiral in from $6R_{Mars}$ to $2.9R_{Mars.}$ Phobos has entered the Roche's zone at Roche's limit = $3.2R_{Mars}$ and Phobos is under tidal stress. It can undergo material's failure or if it survives it will crash into Mars in 10 My according to KM. Deimos tumbled long of inner Clarke's orbit hence as shown in Figure 3 it is launched on expanding spiral orbit but since its mass ratio is 10^{-8} its Time constant of evolution is in Gy and in 4.5Gy it has evolved from $6R_{Mars}$ to $6.9R_{Mars}$ which is snail's pace on account of vanishingly small mass ratio and subsequent weal tidal interaction.

Corroboration and validation of Architectural Design Rules as illustrated in Figure 2 and 3 by advanced simulation done by Canup and Salmon (2018).

Canup and Salmon , 2018, used hybrid N-body model of moon accretion, that includes a full treatment of moon-moon dynamical interactions, to identify the circum-martian impact generated disk of mass $M_{disk} = 10^{18}$Kg which will produce the tiny moons Phobos and Deimos at the edge of the disk. The optimum disk of $M_{disk} = 10^{18}$Kg extends radially to 6Mars radii so that the tiny moons formed at the edge are placed at the orbit with radius $6R_{Mars}$. Canup and Salmon have found that integrating back in time for 4.5Gy from the present orbit of $6.9R_{Mars}$and $2.9R_{Mars}$ for Deimos and Phobos respectively they interpolated back to $6R_{Mars}$ which is synchronous orbit in their jargon but is really triple synchrony orbit also known as inner Clarke's orbit in Author's jargon developed for KM. (Peale & Canup 2015). A less massive disk is too compact to place Deimos at $6.9R_{Mars}$ orbit. A disk mass greater than 10^{18}Kg is too massive and they produce massive moons which get lost through death spirals. Once disk mass $M_{disk} = 10^{18}$Kg is constrained as the optimum disk mass, an optimum impactor for generating this size disk is investigated. Through an advanced Smooth Particle Hydrodynamics (SPH) integrator simulations of impacts are carried out. It is determined that Vesta-to-Ceres sized impactor will create the optimum sized disk. Thus this advanced simulation by Canup and Salmon(2018) establishes that both tiny moons originate in the region between $5R_{Mars}$ and $7R_{Mars}$ near $a_{G1} = 6R_{Mars}$ but Phobos lying on the short side and Deimos lying on the long side of $a_{G1} = 6R_{Mars}$. Thus the simulation results corroborate the Architectural Design Rules. In effect the advanced simulation of Canup and Salmon (2018) validates the Architectural Design Rules proposed by the Author in 35th Scientific Assembly-2004.

Mars – Phobos - Deimos system analysis based on Kinematic Model of tidally interacting binaries

The scientists working in the field of tidally interacting binaries have always used Elasto-Viscous model of tidally interacting binaries (see supplementary notes Sharma 2023A). Elasto-viscous model requires Love Number and Quality Factor which depend upon density, rigidity, viscosity and rate of periodic forcing. These parameters are known with large uncertainties for different Planets and their Satellites and hence their Tidal Evolutionary History will be determined with equal uncertainty by Seismic Model based analysis. Hence the Author took the alternative route of Kinematic Modelling of tidally interacting binaries. In this analysis only the globe-orbit parameters and the age of the tidally evolving binary should be known with high level of confidence. Once this is known then the tidal evolutionary history can be determined with high degree of reliability and accuracy which can be observationally verified.

Kinematic Model of tidally interacting binaries is given in full details in Sharma (2011) and also in supplementary materials Sharma 2023B. The high lights of KM are as follows:

$$\begin{aligned}&\omega(spin\ angular\ velocity\ of\ the\ primary)\\&=\Omega(orbital\ angular\ velocity)\\&=\Omega'(spin\ angular\ velocity\ of\ the\ secondary \quad 4\end{aligned}$$

From the rigorous analysis of Earth-Moon System in my personal communication http://arXiv.org/abs/0805.0100and in my two papers (Sharma *et al.* 2009 and Sharma 2011) the following scenario has emerged:

In section 2 we have studied the evolutionary history of tidally interacting binaries. Throughout this tidal evolutionary history the Total Angular Momentum is conserved hence we have the following Conservation of Momentum equation:

$$\begin{aligned}J_T = C\omega + \left(m^* a_{present}^2 + I\right)\Omega = [C + (m^* a_{G1}^2 + I)]\Omega_{aG1}\\ = [C + (m^* a_{G2}^2 + I)]\Omega_{aG2} \quad 5\end{aligned}$$

In (5):

C = Moment of Inertia of the Primary around its spin axis.
I = Moment of Inertia of the Secondary around its spin axis.
And m*= reduced mass of the secondary = m/(1+m/M) where m = the mass of the secondary and M= mass of the primary.

From Kepler's Third Law:

$$\Omega_{aG1} = \frac{B}{a_{G1}^{3/2}} \ and \ \Omega_{aG2} = \frac{B}{a_{G2}^{3/2}} \ where \ B = \sqrt{G(M+m)} \qquad 6$$

Substituting (6) in (5) we get:

$$J_T = C\omega + \left(m^* a_{present}^2 + I\right)\Omega = \left[C + (m^* a_{G1}^2 + I)\right]\frac{B}{a_{G1}^{3/2}}$$

$$= \left[C + (m^* a_{G2}^2 + I)\right]\frac{B}{a_{G2}^{\frac{3}{2}}} \qquad 7$$

Solving (7) we get the two roots of the Binary System namely a_{G1} and a_{G2}. In classical Newtonian Mechanics also two triple synchrony orbits exist as shown in supplementary materials Sharma 2024. Hence I call this is Newtonian Kinematic Model.

$$J_T = C\omega + \left(m^* a_{present}^2 + I\right)\Omega = \left[C + (m^* a_{G1}^2 + I)\right]\Omega$$

$$= \left[1 + (\theta_2' \times a_{G1}^2 + \theta_1)\right]\frac{CB}{a_{G1}^{3/2}} \qquad 8$$

$$In\ Eq.6,\ \theta_1 = \frac{I}{C} \ and \ \theta_2' = \frac{m^*}{C};$$

Solution of (8) gives the two Triple Synchrony Orbits defined as Clarke's Orbits:

$$Inner\ Clarke's\ Orbit = a_{G1} \ and \ Outer\ Clarke's\ Orbit = a_{G2}$$

Rewriting (8) we get:

$$\frac{J_T}{C\Omega} = \left[\frac{\omega}{\Omega} + \left(\left(\frac{m^*}{C}\right)a_{present}^2 + \frac{I}{C}\right)\right]$$

$$= \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \qquad 9$$

Substituting Kepler's Third Law in (9) we get :

$$\left(\frac{J_T}{CB}\right)a^{\frac{3}{2}} = \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \qquad 10$$

Rearranging the terms of (10) we get:

$$\frac{\omega}{\Omega} = \left(\frac{J_T}{CB}\right)a^{\frac{3}{2}} - (\theta_2' a^2 + \theta_1) = Aa^{\frac{3}{2}} - Fa^2 \qquad 11$$

$$where\ A = \frac{J_T}{BC} \ and \ F = \left(\theta_2' + \frac{\theta_1}{a^2}\right)$$

From Classical Mechanics the Synchronous Orbit is the same as the Inner Clarke's Orbit calculated in Kinematic Framework. In Classical Mechanics, the synchronous orbit is defined as:

$$a_{sync}^{3/2}\Omega_{orb} = a_{sync}^{3/2}\omega_{primary} = B \qquad 12$$

In Classical Mechanics there is no outer Clarke's Orbit. For vanishingly small values of 'q' where q = m/M, the outer Clarke's Orbits are too large to be perceptible but in Earth-Moon system or in Pluto-Charon system where mass ratios are 1/81 and 1/8 respectively. Hence the outer Clarke's Orbit are finite and perceptible.

The probable origin and ages of Phobos and Deimos: Phobos and Deimos are the two moons of Mars. They were discovered by Asaph Hall in 1877. Grey coloured Phobos and Deimos are quite unlike ruddy, pink-skied planet Mars. The two natural satellites are pitted and like drought-state potato. Their surfaces are seared by meteorites and raked by solar wind. They have much lighter density and are probably formed of carbonaceous chronditic material found in outer part of the asteroid belt (Burns 1978). The central force of these lilliputian natural satellites are weak hence the constituent materials have not undergone compaction. These natural satellites have escaped the deeper trauma of heating and inner shifting that have occurred in the formation of Planets. There is evidence that a large object collided with Phobos sometime in the past causing Stickney Crater. This could have weakened the internal structure. This weakening will result in tidal pulverization of Phobos within Roche's limit. NASA announced on 10th November 2015 that grooves have been found on Phobos and this could signal the end of the small body.

The spectral studies of the two satellites match them with carbonaceous asteroids (Rivkin (2002), Burns(1992)). This indicates that they could be captured. However, Phobos and Deimos have circular (with eccentricities ≤0.015) orbits. Their orbital planes are coplanar with Martian equatorial plane (with inclinations ≤1.8°), prograde, well inside the Hill sphere of Mars. This orbital configuration indicates that the two moonlets have accreted from a debris ring (Cameron *et al. (*1976), Charnoz *et al. (*2010), Crida and Charnoz (2012)). Tidal locking with Mars has led many researchers to favour a giant-impact origin for Phobos and Deimos over capture (Caddock (2011), Rosenblatt and Charnoz (2012), Citron *et al. (*2015) Canup and Salmon(2016)). A ~2, 000 km diameter body at an eon > 4.3 Gyr ago made a direct impact with Mars.

This direct impact resulted in the formation of the hemispherical dichotomy seen on Mars (Leone et al.(2014), Nimmo *et al. (*2008), Andrews-Hanna *et al.* (2008), Marinova *et al.* (2008)). Numerical models of giant impacts that could form the dichotomy show that the collision could have ejected as much as 10^23g of debris into Martian orbit (Citron *et al. (*2015), Marinova *et al.* (2011)), of which some portion should form a debris ring of mixed composition orbiting the planet. A debris ring composed of a mixture of impactor and Martian material that proceeds to accrete satellites may explain both the physical and orbital characteristics of the Martian satellites (Craddock (2011)). Deimos, lying outside the synchronous orbit of Mars (located at~6 Mars radii) is subject to tidal torques that cause its semi-major axis to increase over time. Phobos, lying inside the synchronous orbit is gradually evolving inwards, towards Mars (Murray and Dermott, 1999].

Hence by general consensus of the older researchers, the capture origin is discarded (Goldreich 1963; Singer, 1970; Lambeck1979; Szeto 1983). By the study of Mars impact ejecta in the regolith of Phobos it has been concluded that the bulk concentration of Mars-like material in the regolith of Phobos greatly exceeds the upper predicted range of 1250 ppm for Mars ejecta in the regolith of Phobos (Ramsley & Head III 2013). This indicates an interior of Phobos that has a mineralogy similar to that of Mars. This may provide strong evidence that Phobos originated either from a primordial impact on Mars or co-accreted with Mars (Werner, 2008; Andert *et al.,* 2010; Craddock, 2011). Because of these new researches the Author assumes the age of Phobos and Deimos to be 4.5 Gy.

Kinematic Analysis of Mars-Phobos-Deimos

Table 2. Globe and Orbit Parameters of Mars-Phobos-Deimos

Parameters	Mars	Phobos	Deimos	Source
Mass(Kg)	0.64174×10^{24}	10.7046×10^{15}	2.24888×10^{15}	Ref 1,2
GM(Km^3/s^2)	0.042828382×10^{6}	$(7.14\pm0.19)\times10^{-4}$	$(1.5\pm0.11)\times10^{-4}$	Ref 2
Volumetric Mean Radius Or Median Radius ($\times10^3$ m)	3389.5	11.2	6.1	Ref.1
Flattening	0.00589	irregular	irregular	Ref 1
Mean Density(Kg/m^3)	3933	1900	1750	Ref 1
Moment of Inertia($I/(MR^2)$)	0.366	0.4	0.4	Ref 1
Sidereal Spin period	24.6229h	0.31891d	1.26244d	Ref 1
Sidereal Orbital period(d)	-	0.31891d	1.26244d	Ref 1
a*(semi-major axis)($\times10^6$m)	-	9.378	23.459	Ref 1
Orbital eccentricity	-	0.0151	0.0005	Ref 1
Orbital inclination w.r.t. The equatorial plane of Mars(deg)	-	1.08	1.79	Ref 1
Roche's Limit a_R† (Km)		10,885.1	11,187.6	

*Mean Orbital Distance from the center of Mars.

† $a_R = (16\rho_{Mars}/\rho_{Phobos})^{1/3}\times R_{Mars}$;

Table 3. Derived Kinematic Parameters needed in Kinematic Model

Parameters	Mars	Phobos	Deimos	Source
Moment of Inertia around the spin axis(Kg-m^2)	C= 2.69843×10^{36}	I_1= 5.37114×10^{23}	I_2= 3.34723×10^{22}	Calculated
Reduced Mass $m^*=m/(1+m/M)$_($\times10^{15}$Kg)		10.704599821	2.24887999212	calculated
$A = J_T/(B\times C)$ ($\times10^{-11}$)		1.0834199115213353	1.0834190992037116	
Θ_1 (I/C)_($\times10^{-14}$)		19.9047	1.24044	calculated
$F = \Theta_2'(m^*/C)$_($\times10^{-22}$)		39.6697	8.33403	calculated
$B=\sqrt{[G(M+m)]}$_($\times10^6$)$m^{3/2}/s$		6.54248	6.54248	calculated
Present Spin Angular Velocity of Mars(radians/s)	7.08824×10^{-5}			
Present Orbital/Spin Angular Velocity of Phobos(radians/s)	2.28033×10^{-4}			
Present Orbital/Spin Angular Velocity of Deimos(radians/s)	5.76044×10^{-5}			
J_T(total ang. momentum) ($\times10^{32}$Kg-m^2/s)		1.912715482	1.9127140479	Calculated from (5)
Inner Clarke's Orbit(m) a_{G1}		2.04238×10^{7}	2.04238×10^{7}	From (8)
Outer Clarke's Orbit(m) a_{G2}		7.4589×10^{18}	1.68998×10^{20}	From (8)
LOM/LOD eq.11 at current 'a'		0.311144	1.23101	Calculate from eq. 11
LOM/LOD by observation		0.310842	1.2305	observed

Reference 1.http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html

Reference 2. Bills, Bruce G., Neumann, Gregory A., Smith, David E. and Zuber, Maria T. "Improved estimate of tidal dissipation within Mars from MOLA observations of the shadow of Phobos", *JOURNAL OF GEOPHYSICAL RESEARCH*, 110, E07004, doi:10.1029/2004JE002376, 2005

Inspection of the Table 2 clearly establishes that Phobos and Deimos are tidally locked with Mars. They present the same face to Mars all the time. The two satellites are moving in nearly circular orbits and are in nearly coplanar orbital plane. The orbital plane of the natural satellites are coplanar with the equatorial plane of Mars

Calculation of the spiral trajectory of Phobos and Deimos: For the calculation of the spiral trajectory we need the radial velocity of recession in case of super-synchronous configuration and velocity of approach in case of sub-synchronous configuration. The radial integration of the reciprocal of radial velocity gives the non-Keplerian Transit time from its inception to the present orbit. This transit time should be equal to the age of the secondary. The starting point of this radial integral will be the tidal torque.

The Tidal Torque of Satellite on the Planet and of Planet on the Satellite = Rate of change of angular momentum hence

$$Tidal\,Torque = T = \frac{dJ_{orb}}{dt} \quad 13$$

But Orbital Angular Momentum:

$$J_{orb} = m^* a^2 \times \frac{B}{a^{3/2}} = m^* B\sqrt{a} \quad 14$$

Time Derivative of (14) is:

$$T = \frac{dJ_{orb}}{dt} = \frac{m^* B}{2\sqrt{a}} \times \frac{da}{dt} \quad 15$$

In super-synchronous orbit, the radius vector joining the satellite and the center of the planet is lagging planetary tidal bulge hence the satellite is retarding the planetary spin and the tidal torque is BRAKING TORQUE [see supplementary materials Sharma 2023A].

In sub-synchronous orbit, the radius vector joining the satellite and the center of the planet is leading planetary tidal bulge hence the satellite is spinning up the planet and the tidal torque is ACCELERATING TORQUE. [see supplementary materials Sharma 2023A].

I have assumed the empirical form of the Tidal Torque as follows:

$$T = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \quad 16$$

(16) implies that at Inner Clarke's Orbit and at Outer Clarke's Orbit, tidal torque is zero and (17) implies that radial velocity is zero and there is no spiral-in or spiral-out.

At Triple Synchrony, Satellite-Planet Radius Vector is aligned with planetary tidal bulge and the system is in equilibrium. But there are two roots of ω/Ω=1: Inner Clarke's Orbit and Outer Clarke's Orbit. As already shown [see supplementary Sharma 2024] in Total Energy Profile, Inner Clarke's Orbit a_{G1} is unstable equilibrium state and Outer Clarke's Orbit a_{G2} is stable equilibrium state. In any Binary System, secondary is conceived at $a_{G1.}$ This is the CONJECTURE assumed in Kinematic Model. From this point of inception Secondary may either tumble short of a_{G1} or tumble long of $a_{G1.}$ If it tumbles short, satellite gets trapped in Death Spiral and it is doomed for destruction. If it tumbles long, satellite gets launched on an expanding spiral orbit due to gravitational sling shot impulsive torque which quickly decays. After the impulsive torque has decayed, the satellite coasts on it own toward final lock-in at $a_{G2.}$

Equating the magnitudes of the torque in (15) and (16) we get:

$$\frac{m^*B}{2\sqrt{a}} \times \frac{da}{dt} = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \qquad 17$$

Rearranging the terms in (17) we get:

$$V(a) = Velocity\ of\ recession$$

$$= \frac{2K}{m^*B} \times \frac{\frac{1}{a^Q}\left[Aa^2 - Fa^{2.5} - \sqrt{a}\right]m}{s} \qquad 18$$

The Velocity in (18) is given in m/s but we want to work in m/y therefore (16) R.H.S is multiplied by 31.5569088×10^6s/(solar year).

$$V(a) = \frac{2K}{m^*B} \times \frac{1}{a^Q}\left[Aa^2 - Fa^{2.5} - \sqrt{a}\right] \times 31.5569088 \times \frac{10^6 m}{y} \quad 19$$

In (19) 'a' refers to the semi-major axis of the evolving Satellite. There are two unknowns : exponent 'Q' and structure constant 'K'. Therefore two unequivocal boundary conditions are required for the complete determination of the Velocity of Recession.

First boundary condition is at a = a_2 which is a Gravitational Resonance Point where ω/Ω = 2 (Rubicam 1975),

i.e. $(Aa^{3/2} – Fa^2) = 2$ has a root at $a_{2.}$

In Mars-Phobos case, $a_2 = 3.24207\times10^7$ m.

At a_2 the velocity of recession maxima occurs. i.e. $V(a_2) = V_{max}$ Therefore at a = a_2, $(\delta V(a)/\delta a)(\delta a/\delta t)|_{a2} = 0$.

On carrying out the partial derivative of V(a) with respect to 'a' we get the following:

$$At\ a_2, (2-Q)A \times a^{1.5} - (2.5-Q)F \times a^2 - (0.5-Q) = 0 \quad 20$$

Root of (20) gives the exponent term 'Q'.

Now structure constant (K) has to be determined. This will be done by trial error so as to get the right age of Phobos i.e. 4.5Gy.

We will assume the age of Mars and Deimos as 4.5Gy as already mentioned in Section 5.1. The Transit Time from a_{G1} to the present 'a' is given as follows:

$$Transit\ Time = \int_{a_{G1}}^{a} \frac{1}{V(a)} da \qquad 21$$

The results of the calculations of spiral trajectory for Phobos (collapsing Spiral) and for Deimos (expanding spiral) are tabulated in Table 4 and Table 5 respectively.

Table 4.Timeline of the gravitational runaway collaping spiral orbit of Phobos

#	'a'(×10^6m)	'a'/R_{Mars}	Time (after the birth)	comment
1	20.423585*	6.02555	0	Phobos is conceived short of a_{G1}
2	20.42358	6.02554359	12.381My	
3	20.42357	6.02554064	36.49My	
4	20.42356	6.02553769	57.709My	
5	20.42355	6.02553474	78.9My	
6	20.4235	6.02552	172.251My	
7	20.421	6.02478	1.23685Gy	
8	20.42	6.02449	1.37794Gy	
9	20.415	6.02301	1.7643Gy	
10	20.41	6.02154	1.9707Gy	
11	20.4	6.01859	2.22018Gy	
12	20.3	5.9808	2.96993Gy	
13	20.2	5.95958	3.23511Gy	
14	20.1	5.93008	3.39826Gy	
15	20.0	5.90058	3.51549Gy	
16	19	5.60555	4.01137Gy	
17	18	5.31052	4.19829Gy	
18	17	5.01549	4.30217Gy	
19	15	4.42543	4.41131Gy	
20	13	3.83537	4.46258Gy	
21	12	3.54035	4.47745Gy	
22	11	3.24532	4.48791Gy	
23	10	2.95029	4.49518Gy	
24	*9.378*	*2.76678*	*4.49851Gy*	*PRESENT orbit, current altitudinal decay rate=-21.1473cm/y*
25	9	2.65526	4.50017Gy	future spiral path
26	8	2.36023	4.50351Gy	future spiral path
27	7	2.0652	4.50566Gy	future spiral path
28	6	1.77017	4.5076Gy	future spiral path
29	5	1.4714	4.50777Gy	future spiral path
30	4	1.18	4.50818Gy	future spiral path
31	3.3895	1	4.50831Gy	Surface of Mars-DOOMSDAY 9.79815My from the present.

*Birth orbit of Phobos on the short side of $a_{G1} = 2.04238\times10^7$m

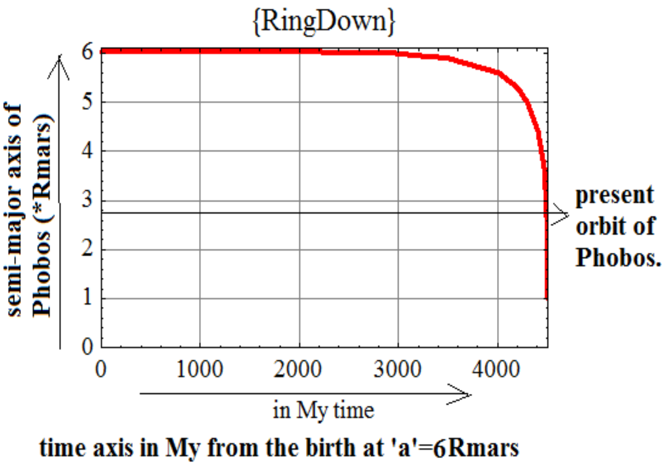

Figure 4. 1-D profile of collapsing spiral orbit of Phobos. Chirp signal is clearly identifiable towards the end when in-spiral rate exponentially increase. [Courtesy:Author]

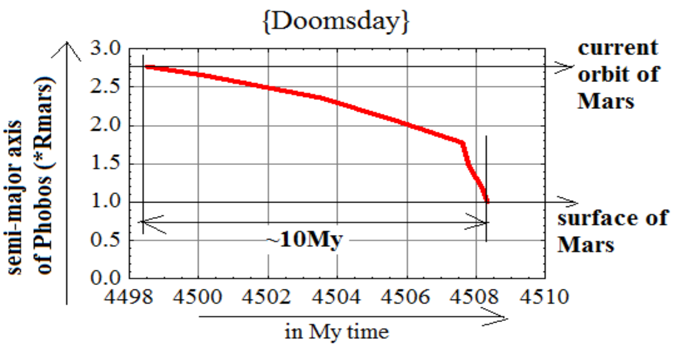

Figure 5. Final plunge of Phobos into the surface of Mars during the futuristic spiral-in of Phobos-Mars over a time span of ~10My. [Courtesy:Author]

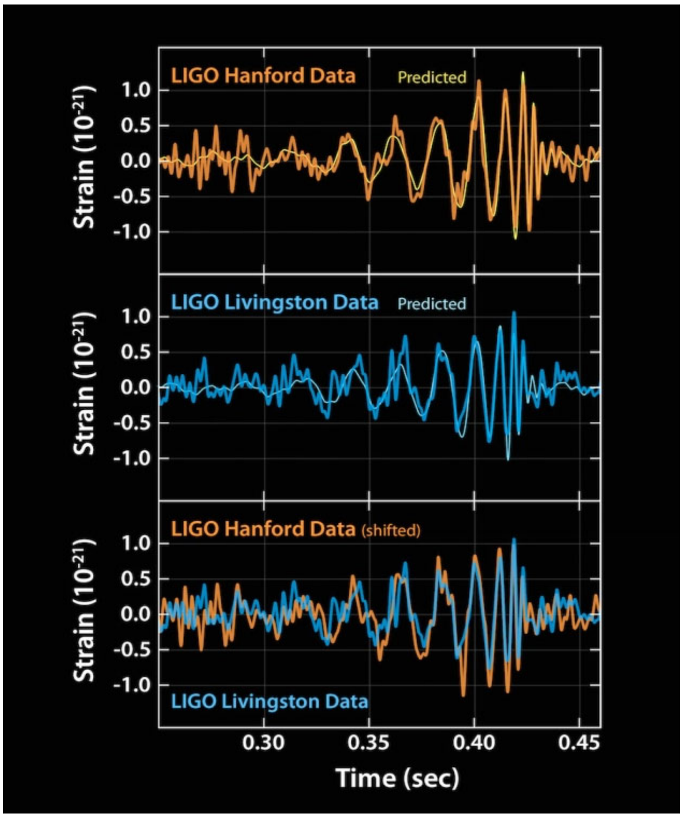

Figure 6. The inspiral and merger of Black Hole binaries and chirp signal detected simultaneously at LIGO observatory at Hanford and Livingston. {Courtesy: Ligo announces gravitational wave detection –in pictures, Chris Maddalni.and Lauren Morello, 11th February 2016, Nature News, doi:10.1038/nature.2016.19368]

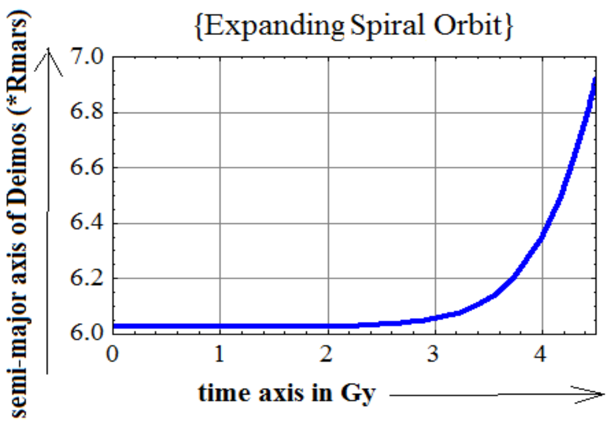

Figure 7. Gradually expanding spiral orbit of Deimos over the life-time of 4.5Gy. [Courtesy:Author]

Table 5. Timeline of the gradually expanding spiral orbit of Deimos

#	‘a’(×10^6m)	‘a’/R_{Mars}	Time (after the birth)	Comment
1	20.42381*	6.02561144	0	Deimos is conceived long of a_{G1}
2	20.42382	6.02561440	76.29My	
3	20.42383	6.02561735	140.1My	
4	20.42384	6.02520298	194.93My	
5	20.42385	6.02562325	243My	
6	20.42386	6.02562620	285.802My	
7	20.42388	6.02563210	359.474My	
8	20.42389	6.02563505	391.68My	
9	20.4239	6.0256380	421.433My	
10	20.424	6.0256675	636.08My	
11	20.4241	6.0256970	774.139My	
12	20.4242	6.0257265	875.821My	
13	20.4243	6.0257560	956My	
14	20.4244	6.02579	1.02322Gy	
15	20.4245	6.02582	1.08024Gy	
16	20.4246	6.02584	1.12999Gy	
17	20.4247	6.02587	1.17411Gy	
18	20.4248	6.0259	1.21375Gy	
19	20.4249	6.02593	1.24974Gy	
20	20.425	6.02596	1.2827Gy	
21	20.426	6.02626	1.51427Gy	
22	20.427	6.02655	1.6587Gy	
23	20.428	6.02685	1.76394Gy	
24	20.429	6.02714	1.84678Gy	
25	20.43	6.02744	1.9151Gy	
26	20.44	6.03039	2.28941Gy	
27	20.45	6.03334	2.4774Gy	
28	20.46	6.03629	2.60407Gy	
29	20.47	6.03924	2.6998Gy	
30	20.48	6.04219	2.77681Gy	
31	20.49	6.04514	2.84128Gy	
32	20.5	6.04809	2.89673Gy	
33	20.6	6.07759	3.22937Gy	
34	20.7	6.1071	3.41024Gy	
35	20.8	6.1366	3.53627Gy	
36	20.9	6.1661	3.63369Gy	
37	21	6.1956	3.71353Gy	
38	21.5	6.34312	3.984786Gy	
39	22	6.49063	4.16162Gy	
40	22.5	6.63815	4.29759Gy	
41	23	6.78566	4.41081Gy	
42	23.1	6.81516	4.43154Gy	
43	23.2	6.84467	4.45175Gy	
44	23.3	6.87417	4.47Gy	
45	23.4	6.90367	4.49Gy	
46	23.459	6.92108	4.5Gy	

*Birth orbit of Deimos on the long side of $a_{G1} = 2.04238\times10^7$m

Study of Figure 4 and Figure 5 clearly establishes that just as there is a chirp signal shown in Figure 6, accompanying in-spiraling relativistic pairs (Black hole pairs/neutron star pais), there is a chirp signal of non-relativistic pairs (Mars-Phobos pair) but for entirely different reasons. Chirp signal means increasing rate of in-spiraling binary components with time. They spiral-in faster. This is exactly what is happening here in Figure 4. Phobos in-spiral is gradual but progressively it increases until it plunges into Mars.

In both cases there is a runaway phenomena but the positive feedback loop has different physics. Mars-Phobos is launched on gravitationally runaway collapsing spiral orbit because of rotational energy transfer cycle in a positive feed-back loop. Whereas Black Holes pair or Neutron Star pair get launched on a runaway spiral merger because of gravitational wave radiation which were detected recently by LIGO,

Table 6. Kinematic Parameters of the spiral trajectory of Phobos and Deimos

Parameters	Phobos	Deimos
a_2 ($\times10^7$m)	3.24207	3.24207
Q(exponent)	3.5	3.49999
B ($\times10^6 m^{3/2}/s$)	6.54248	6.54248
V_{max}(m/y)	0.00743	0.0087
K(structure constant)[$\times10^{35}$]	2.80961	0.691082

Table 7.The Transit Time, Dooms day expected for Phobos and Approach Velocity for Phobos and Recession Velocity for Deimos

Parameters	Phobos	Deimos
Time Constant of evolution(τ)^1 [$\times10^{22}$y)	0.100389	1.94251
Evolution Factor(€)^2	-1.48088×10^{-12}	1.796×10^{-14}
Transit time (age of the binary system)	4.50293Gy	4.50734Gy
Expected Dooms day of Phobos	9.79742My in future	Not applicable
Radial Velocity(Approach or Recession)	-0.211473m/y	+0.005313m/y

Time Constant of Evolution = $\tau = (a_{G2} - a_{G1})/ V_{max}$;
Evolution Factor = € $= (a- a_{G1})/ (a_{G2} - a_{G1})$

Transit Time from the current orbit of Phobos 'a' = 9378Km to the surface of Mars at a = 3389.5Km takes ~10My. Hence doomsday is predicted at 10My from now. But infact Phobos is already within Roche's Limit of 10, 886 Km and is still intact. This implies that that Phobos is hard enough to withstand tidal pulverization but it is under considerable tidal stress. If structural failure occurs under considerable tidal stress due to crack formation, which has already started as reported by NASA (Hurford *et al.* 2016), then any time now Phobos will be tidally pulverized into a Martian ring of dust. The analysis based on Kinematic Model but assuming the altitude decay rate as derived by Johnson(1972) and Bills(2005) based on Seismic Model give the same time for dooms day as the estimation based on Seismic Model but give technically untenable age of Phobos. These results are tabulated in Table 8

Table 8. Transit Time and Dooms day estimate by Kinematic Model assuming the altitude decay rate as calculated by Witasse *et al. (*2013) and Bills (2005)

Altitude decay rate	-0.018m/y (Witasse *et al.* 2014)	-0.0398858m/y (Bills 2005)
Transit Time(Age) from Kinematic Model	52.9027Gy	23.8744Gy
Dooms day estimate from Kinematic Model	115.105My	51.9455My

In Kinematic Model analysis, if the altitude decay rate is assumed as calculated by Witasse *et al. (*2013) and Bills *et al. (*2005) then we arrive at the same dooms day time table as estimated by Witasse *et al.* (2013) and Bills *et al. (*2005) based on Seismic Model which is 100My and 50 to 30 My respectively but the transit time is 53Gy and 24Gy respectively from Kinematic Model. The transit time is inordinately large. Hence Seismic Model does not seem to be giving realistic results.

Phobos Orbital Mean Motion Resonance 2:1 with Deimos and its consequences: Tidal evolution of Phobos from inner Clarke's Orbit a_{G1} would place the satellite in its current location after ~4.5 Gyr; but as Phobos is spiraling inward it will cross 2:1 Mean Motion Resonance orbit with respect to Deimos where Deimos: Phobos orbital periods are in 2:1 ratio. Deimos is at 23459Km and this orbital radius will change very little as the time constant of evolution is inordinately large at 2×10^{22}y but as Phobos spirals past the orbital radius of 14778Km, 2:1 MMR crossing occurs. This orbital resonance is expected to raise Deimos eccentricity to a much a higher value then present eccentricity of 0.0005 observed (Murray and Dermont (1999), Yoker (1982)). But this does not happen. Because of very small masses of Phobos and Deimos, the two donot mutually interact (Veverka and Burns (1980)). Being deep in Hill's sphere the planetary perturbations due to other planets (other than Mars) has negligible effect.

The final fate of Phobos and Deimos: Because Deimos is in super-synchronous orbit and because Deimos to Mars mass ratio is 10^{-9} hence its time constant of evolution is inordinately large and for all practical purposes Deimos has no evolutionary history and it remains stay-put at 23459Km orbital radius but Phobos is trapped in a death spiral rapidly spiraling in at a radial velocity of 21cm per year presently and expected to plunge into Mars at glancing angle collision in 10My from now. But even before head-on collision, material failure can occur and Mars may be tidally pulverized into a Martian ring.

DISCUSSION

This study has invoked Kinematic Model to study Mars-Phobos and Mars-Deimos and correctly derived two Clarke's orbits in case of Mars-Phobos and Mars-Deimos. In case of Mars-Phobos, this study has correctly arrived at 10My as the remaining time span of Phobos existence. This study also asserts that Phobos is under considerable tidal stress and at any time within this time span structural failure can lead to tidal pulverization and its spread into a Martian Ring. Black and Mittal (2016) put this time span at 20My to 40My whereas Hesselbrock and Minton (2017) put this time pan at 70My. Unlike this study both groups have used elasto-viscous model of tidally interacting binaries with all its uncertainties in Quality Factor and Love Number. For Hessebrock and Minton (2017) eccentricity of Deimos being accentuated by 2:1 MMR crossing by Phobos at a=14778.25Km during its spiral –in journey has been a major concern. But as we have seen scale of Phobos and Deimos mass makes mutual interaction insignificant (Veverka and Burns (1980)) hence the question of eccentricity being accentuated by orbital resonance does not arise.

Conclusion

The results in this paper seems to be reasonable considering the fact that Phobos is in a gravitationally runaway in-spiral path but the ultimate validation or invalidation of these results will come from future Interplanetary Laser Ranging Missions(ILRM) notably from Phobos Laser Ranging Mission(PLRM) [Appendix A]

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Conflict of Interest: I have no conflict of interest whatsoever with anybody.

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APPENDIX A.

Interplanetary Laser Ranging (Turyshev *et al.,* 2010)
(All references in the main text reference)

With recent successful Laser Transponder Experiments conducted with MLA(Mercury Laser Altimeter)[1] and MOLA(Mars Orbiter Laser Altimeter)[2] instruments (Smith *et al.* 2006; Sun *et al.* 2005; Abshire *et al.* 2006; Degnan 2008), Interplanetary Laser Ranging (ILR) is rapidly becoming a mature technology. A mm-level ranger precision over inteplanetary distances is within our reach thus opening a way for significant advances in the tests of gravity on Solar System Scale (Degnan 2007). ILR allows for a very precise trajectory estimation to an accuracy of less than 1cm at a distance of ~2AU. One of these missions being planned is Phobos Laser Ranging (PLR) Mission which is expected to be set up by 2016. In this mission a Laser Ranging Transponder Instrument will be deployed on Phobos. This Transponder will enable measurements of distances from Phobos to Earth with 1-mm accuracy during daily hour long passes (Murphy et.al 2009). Precision Laser Ranging to Phobos could measure the distance between an observatory on the Earth and a terminal on the surface of Phobos to an accuracy of 1-mm in less than 5 minutes of Integration Time.Phobos shows a large secular acceleration in orbital longitude. Recent fits by Bill *et al. (*2005), Lainey *et al. (*2007) and Jacobson (2010) give an acceleration in the forward orbital longitude = a(dn/dt)= $416 m/y^2$. This secular acceleration can be easily detected by PLR giving refined accuracy. The cause of this acceleration is Phobos-raised tides on Mars perturbing Phobos. The tidal bulge in Mars is behind (in time and longitude) Phobos position radius vector as a result Phobos is accelerating Mars spin and in the process sapping energy from the orbit which consequently shrinks by 4cm/y as estimateby Bill *et al.* (2005) and by Ramslay & Head III(2013). Phobos will eventually impact Mars (Efronsky *et al.,* 2010). The most important of the tidal components for the secular acceleration should be the second degree M_2 tide of period 5.55h on Mars. The small eccentricity (0.015) and inclination (1.1°) tend to reduce the influence of other degree 2 tides by ~ 3 order of magnitude or

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[1] MLA is an instrument on MESSENGER Mission. It has conducted successful 2-way experiment in daylight during Earth fly by with satellite 24MKm away. This was in May 2005.

[2] MOLA attempted 1-way LASER Ranging of Mars while orbiting Mars at a distance of 80MKm. In September 2005 on 3rd day it succeeded in LASER Ranging of Mars.

more. The influence of tides of higher degree fall off as even powers of (R/a)=(1/2.76) about an order of magnitude per degree so the degree 3 tide of 3.7h-period on Mars is a small contribution to tidal secular acceleration. Yoder (1982) has placed an upper limit on Phobos k_2(Love Number)/Q= 2×10^{-7}. Which would make dissipation in Phobos a minor contributor of the order 10^{-3} relative to the overall tidal acceleration of Phobos.

Periodic tidal displacement on Phobos might reach 1mm. Meteoric Impact are not a concern for the dynamics of PLR mission. Once the secular acceleration measurement is made by high confidence level in PLR mission the altitude loss can be accurately ascertained and this will provide the ultimate validation or invalidation of the kinematic model and this model based analysis.

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