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Research Article

UNDERSTANDING THE SHAPE OF A CUBE THROUGH ITS NET BY ELEMENTARY SCHOOL STUDENTS

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Abstract

The shape of a cube is one that we often take for granted, assuming it is well-known. Additionally, it is known that in mathematics classes, more and more time is devoted to areas related to numbers, while geometry somehow becomes the one sacrificed in favor of numbers. All these prejudices have obviously left a mark on students' knowledge, as the results of the research conducted in this paper will show. Several research questions were posed within the research part of the paper. Does the choice of tasks and the repetition of patterns in them affect the choice of method when solving tasks that are not a continuation of what students are currently learning in regular classes? Does the choice of a particular solution depend on the student's gender, grade, or year of birth? Is there something unexpected that appears in students' work that could be a significant factor during future testing? The aim of the paper is for the results obtained to serve those who educate students when creating the teaching process.

Keywords: Cube shape, Cube net, Elementary school students, Subject teaching, Mathematics.

INTRODUCTION

Theoretical Framework of the Paper

Mathematical knowledge is often based on a more theoretical foundation. In many cases (such as numerical patterns), inductive reasoning based on experimentation is applied. Artigue and Baptist (2012) believe that the cumulative nature of mathematics poses a challenge when it comes to directly adopting the concept of research from natural sciences, because in natural sciences, a hypothesis (whether of a researcher or a student) is confirmed by experiment, whereas in mathematics, a final conclusion requires proof based on deductive reasoning. Mathematics teaching should be based on how professional mathematicians think, act, and learn. Mathematics educator Mogens Niss explained the understanding of how students develop mathematical knowledge (which is a constant research interest in mathematics education) as follows: "If we understand the possible ways of learning mathematics and the obstacles that prevent these ways for ordinary students, we will better understand what mathematical knowledge, insight, and capabilities are (and are not), how they are created, stored, and activated, and consequently how to promote them." (Niss, 1999). Routine exercises often require students to merely imitate the teacher, which they frequently do without seeing or understanding any logic or meaning in the concepts and procedures they use during problem-solving (Schoenfeld, 1988). Over time, students may indeed start perceiving mathematics as a meaningless set of techniques that must be mastered through imitative practice. This type of teaching deprives students of the experience of many important elements of mathematics, such as solving complex problems, creating coherent knowledge structures, hypothesizing and proving, experimenting with specific cases, and so on. John Dewey is often associated with the expression "learning by doing." He believed that teaching should focus on student activities and the ways students acquire knowledge from them (Dewey, 1902). Dewey (1938) emphasized the potential importance of research and its role in learning and teaching, particularly concerning natural sciences. He largely perceived mathematics as a tool or language for organizing complex information or systematically managing the outcomes of research processes, for instance, the outcomes of students' activities when conducting experiments related to physical laws or biological systems. In 1945, George Pólya published the book "How to Solve It?" which is considered a classic work regarding approaches to problem-solving in mathematics education (Artigue & Blomhøj, 2013). In the book, problem-solving is described as an activity that mathematicians engage in while researching. The emphasis is on the role of problems and the heuristic competencies needed to solve them. Schoenfeld (1992) criticized Pólya's ideas in mathematics teaching, considering that they trivialize the matter too much and do not sufficiently emphasize the key element of developing heuristic competencies in students. He believed that before students start solving problems, a distinction must be made between problems and exercises. Exercises can be solved using known strategies, while problem-solving requires developing or combining methods and knowledge in a new way. Inquiry-based mathematics teaching refers to an approach to mathematics teaching that allows students to engage in activities that lead them to adapt their existing or acquire new mathematical knowledge. It should stem from the students' own activities and efforts. Students should tackle problem-solving or situations themselves because this can lead them to formulate hypotheses, research, and experiment, which leads to the formulation of solutions (Jessen *et al.,* 2017). According to the Fibonacci project, inquiry in natural science teaching often relies on sensory experience (Artigue *et al.,* 2012).Inquiry-based teaching can have two approaches. The first approach is Realistic Mathematics Education (RME), which originates from Hans Freudenthal.

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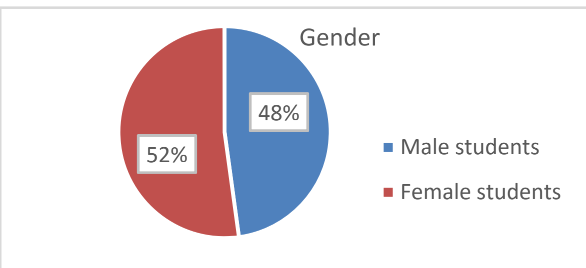
It provides examples of how students' experiences can serve as a starting point for the inquiry process (Freudenthal, 1991). The second approach is called the Theory of Didactical Situations, which originates from Guy Brousseau (Brousseau, 1997). It is based on the idea that students acquire new knowledge when solving problems while adapting to what is called a didactical environment. In inquiry-based teaching, a problem is more than just a task, exercise, or activity. Students need to experiment, theorize about possible solutions, formulate hypotheses and possible strategies, and so on. According to both theories, students are given non-routine problems, and the teacher provides the initial problem, but it is the students who continue to work on it. According to the Theory of Didactical Situations, students adapt to the environment of teaching situations (Brousseau, 1997). According to the Theory of Realistic Mathematics Education, students mathematize the phenomenon that is the subject of the problem both vertically and horizontally (Freudenthal, 1991). Inquiry-based mathematics teaching influences motivation and the development of attitudes towards mathematics as well as the understanding of the importance of mathematics in real life and society (Bruder & Prescott, 2013; Blanchard *et al.,* 2010; Furtak *et al.,* 2012; Hattie, 2009; Minner *et al.,* 2010). However, some experts warn that this type of teaching can lead to better learning only if it is carefully designed and planned (Hofstein & Lunetta, 2004; Woolnough, 1991). Artigue and Blomhøj (2013) state that to describe, research, or understand many phenomena from everyday life, mathematics can be used in combination with logic, and this is a good basis for inquiry-based mathematics teaching. Problem-solving is considered an activity in which students are expected to participate. It implies that students use previously acquired knowledge, intuitions, undefined understandings, and hypotheses to explore and understand the problem. By experimenting with new and previous knowledge, students develop new insights. The problem-solving process is driven by students' mathematical creativity and curiosity, which further develop as the problem is solved. The teacher should guide students in this process not by giving answers but by being an experienced co-researcher who asks questions and thus stimulates the research process. The principles of teaching also address the challenges and dilemmas teachers face regarding when to engage in student activities or refrain from giving answers to students or how to encourage optimal strategies (Schoenfeld, 1992). If a teacher hints that students should consider a special case and mentions it to them, students may perceive it as the only possible way to solve that problem and do so because the teacher said so. In the research process, it is important that there are not too many instructions, but also not too few. Mathematization refers to the entirety of the organizational activity of mathematicians, whether it involves mathematical content and expressions or more naively, intuitively experienced experiences expressed in everyday language. The goal is to offer non-mathematical rich structures to familiarize students with discovering structure, structuring, impoverishing structures, and mathematization. In this way, students can discover poor structures in the context of rich ones in the hope that this approach will function in other (both mathematical and non-mathematical) contexts as well. If one starts with poor mathematical structures, it may mean that one may never reach the rich non-mathematical structures that are actually the appropriate goal (Freudenthal, 1991). Mathematization encompasses: axiomatization (creating an axiomatic mathematical system), formalization (transition from an intuitive to a formal approach), schematization (creating meaningful networks of concepts and procedures), algorithmization (transition from laborious problem-solving to routine problem-solving), modeling (creating schemes that represent, idealize, and simplify other schemes), etc. We distinguish between two directions of mathematization: horizontal and vertical (Treffers, 1987).

Research Framework of the Paper

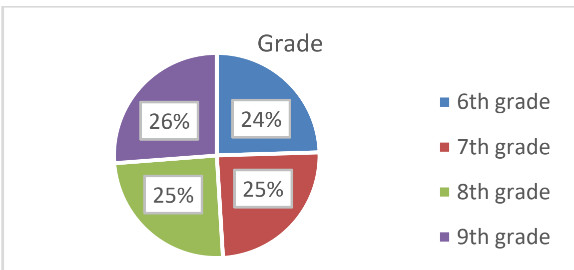
The main idea of this paper was to examine how students from the 6th to the 9th grade of elementary school handle so-called simple/easy/elementary tasks related to cubes and their nets. By analyzing the available mathematics textbooks for that age group, it was observed that there are very few such tasks available to students and that they are mainly based on the same ideas (e.g., the same shape of the net is used most often), and that finding only one solution is generally considered sufficient. This leads to students thinking that anything deviating from what they most commonly encounter is considered an error, and that it is important to find only one solution. From this, one could conclude that in practice, the learning of patterns and forms is nurtured without encouraging students to engage in independently finding new ideas and new ways of solving tasks. All of this raises the question of how much we are actually developing students' logical thinking and how much mathematics teaching is fulfilling its goal (which definitely is not just solving tasks).Since we wanted to obtain more qualitative rather than quantitative data, we randomly selected several elementary schools in the Sarajevo Canton (Bosnia and Herzegovina) and tested students from the 6th to the 9th grade. The students were not pre-prepared nor did they receive any special instructions for the tasks. Strict anonymity was agreed upon, so the data related to the students are known exclusively to the authors of the paper. During the data analysis, it was observed that this anonymity, i.e., the non-conditionality of grades, had a significant impact on the completion of tasks, as well as on the accuracy of what was done.

Sample and Population

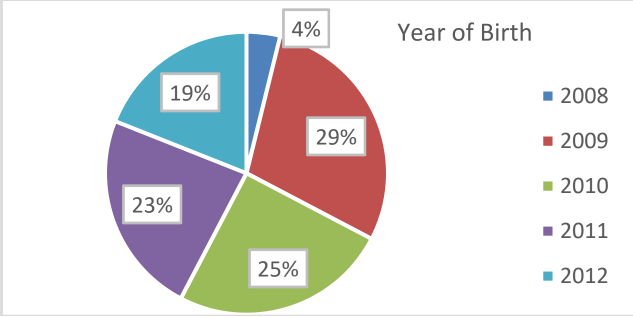
The sample consisted of 416 elementary school students in the Sarajevo Canton (Bosnia and Herzegovina), which is graphically described in detail by gender (Graph 1), by grade (Graph 2), and by year of birth (Graph 3).



Graph 1. Sample distribution by gender



Graph 2. Sample distribution by grade



Graph 3. Sample distribution by year of birth

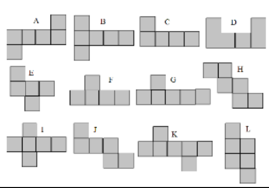
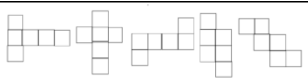
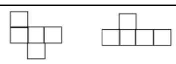
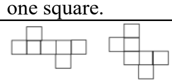
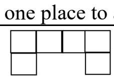
What we can see from the presented data is that the sample is quite balanced in terms of gender and grade. Regarding the year of birth, there are deviations when it comes to the 2008 generation. They are the least represented. After them comes the 2012 generation, which is expected as they are the boundaries of the birth interval from which our observed sample comes. Other years are approximately balanced in terms of the number of respondents. We highlight this to exercise caution when drawing certain conclusions.

Description of the Research Instrument

The research utilized a test that was specifically created for the purpose of this study. It consisted of 5 questions/tasks.

1. **Question 1:** 12 nets were presented, 5 of which were cube nets. The task for the students was to circle the ones that represented a cube net.
2. **Question 2:** 5 nets were presented, 4 of which were cube nets. Students needed to mark the opposite faces with the same labels.
3. **Task 3:** 2 nets were presented. The task was to draw an additional square to form a cube net. There were multiple possible solutions (4 for each net).
4. **Task 4:** 2 nets were presented. The task was to cross out one square on each net to form a cube net. There were multiple possible solutions (2 for each image).
5. **Task 5:** One net was given. By moving one square to another location, the students needed to create a cube net. There were multiple possible solutions (a total of 8 possibilities).

Figure 1. Appearance of the instrument

Task 1: Which of the following figures can be a cube net? Circle those figures.	**Task 2:** In the given cube nets, mark the opposite sides with the same symbol.	**Task 3:** A figure is given. Create a cube net by drawing one square.	**Task 4:** A figure is given. Create a cube net by crossing out one square.	**Task 5:** A figure is given. Create a cube net by moving one square from one place to another.
				

Research Tasks

1. Investigate whether the choice of tasks and the repetition of patterns in them affect the choice of method when solving tasks that are not a continuation of what students are currently doing in regular classes.
2. Investigate whether students more frequently choose a certain solution depending on their gender, grade, or year of birth.
3. Investigate whether anything unexpected will appear during the test-solving process that could be a significant factor for future testing.

Analysis of Task 1

The obtained data were presented using tables, graphs, and images.

Table 1. Frequency of responses by gender

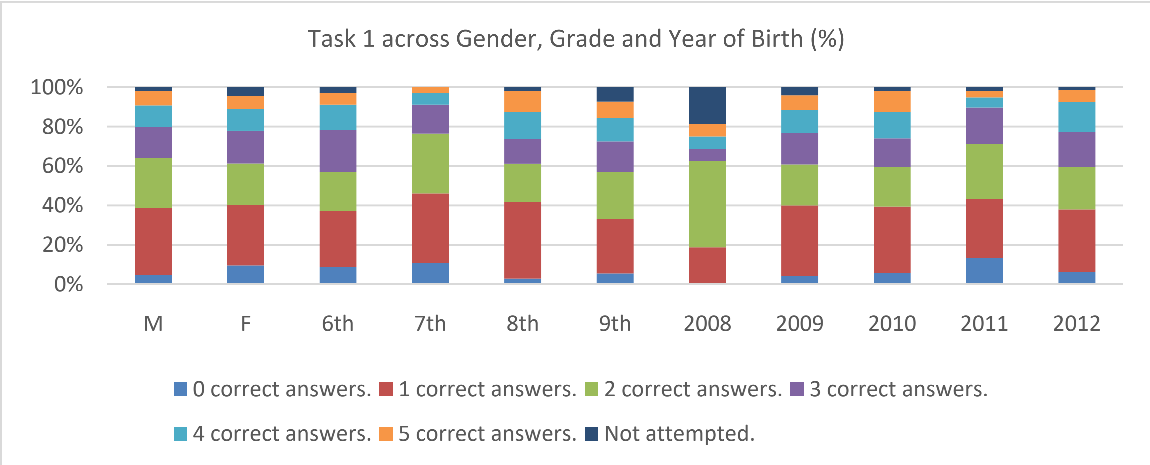
Gender of students * Which of the following figures can be a cube net? Circle those figures. Cross tabulation									
Count									
		Which of the following figures can be a cube net? Circle those figures.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
gender of students	Female	10	74	55	34	24	16	4	217
	Male	19	61	42	33	22	13	9	199
Total		29	135	97	67	46	29	13	416

Table 2. Frequency of responses by grade

Grade * Which of the following figures can be a cube net? Circle those figures. Cross tabulation									
Count									
		Which of the following figures can be a cube net? Circle those figures.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
Grade	6th grade	9	29	20	22	13	6	3	102
	7th grade	11	36	31	15	6	3	0	102
	8th grade	3	40	20	13	14	11	2	103
	9th grade	6	30	26	17	13	9	8	109
Total		29	135	97	67	46	29	13	416

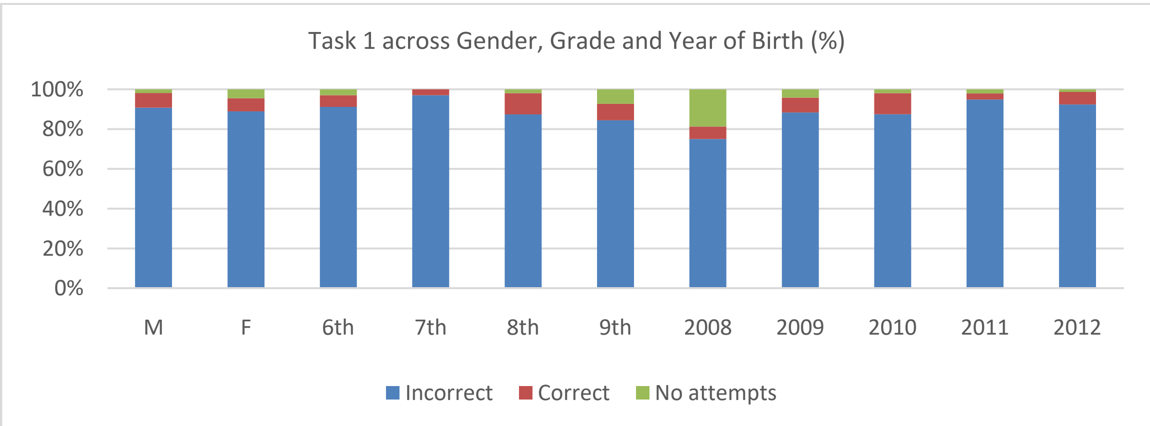
Table 3. Frequency of responses by year of birth

Year of Birth * Which of the following figures can be a cube net? Circle those figures. Cross tabulation									
Count									
		Which of the following figures can be a cube net? Circle those figures.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
year of birth	2008	0	3	7	1	1	1	3	16
	2009	5	43	25	19	14	9	5	120
	2010	6	35	21	15	14	11	2	104
	2011	13	29	27	18	5	3	2	97
	2012	5	25	17	14	12	5	1	79
Total		29	135	97	67	46	29	13	416



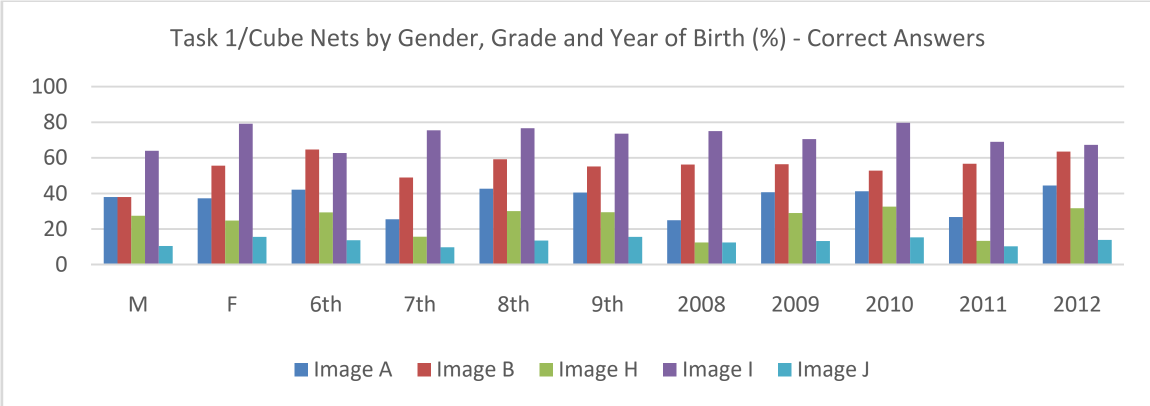
Graph 1. Distribution of all responses for Task 1 across all categories expressed in percentages

When we observe the graphically presented data, we do not see any significant deviations between the columns. However, upon closer examination, the first thing we can notice is that the 2008 generation has no situations with 0 correct answers, but they have the highest number of situations without attempts. In the 7th grade, there are no situations without attempts.



Graph 2. Distribution of responses categorized as correct, incorrect, and no attempts for Task 1 across all categories expressed in percentages

When we observe the data we presented visually, we must be concerned about the extremely high percentage of incorrect answers across all observed categories. Let us note that we examined the correctness of the entire task here. Since the task consisted of 12 nets, 5 of which were cube nets, we can also observe the aspect of task completion in relation to those 5 nets.



Graph 3. Distribution of correct answers for Task 1 across all categories expressed in percentages

On this chart, the percentage of correct answers by all categories and for all 5 cube nets is shown. In the following image, we displayed the order of cube nets from the highest to the lowest percentage. As Net I is definitely the most represented in all mathematics textbooks, we can assume that this factor influenced the students' answers in this question. Since Net B is very similar to Net I, we could assume that this factor also had an influence in this case.

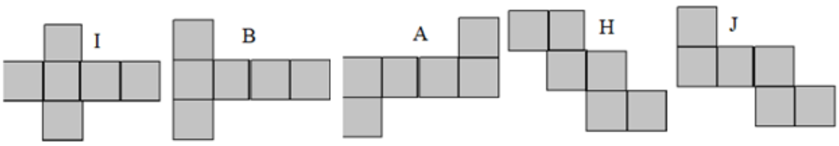


Figure 2. Cube nets from Task 1 arranged by the degree of recognition by students

Thus, one of the assumptions derived from this task is that the frequency of repeating a procedure or method in solving tasks has a significant influence on future task-solving.

Analysis of Task 2

We presented the obtained data using tables, charts, and images.

Table 4. Frequency of responses by gender

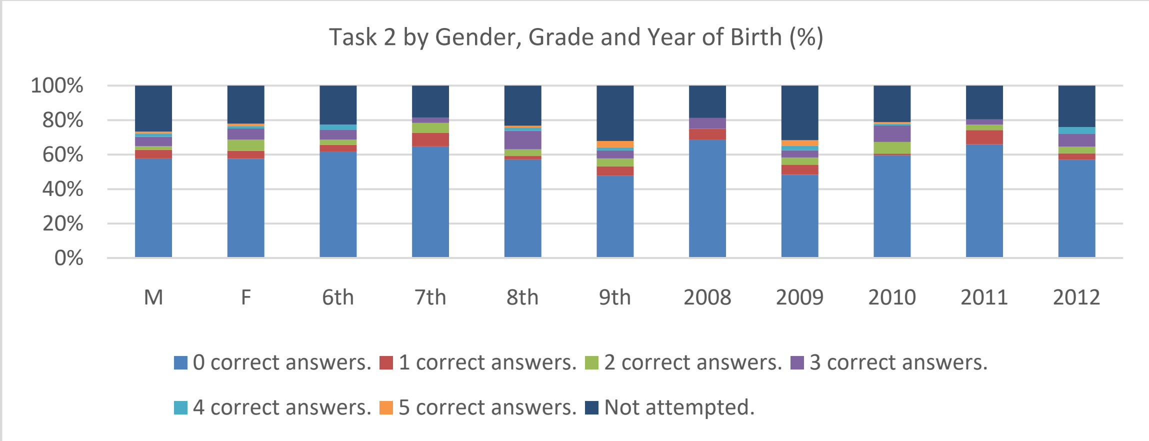
Gender of Students * In the given cube nets, mark opposite faces with the same symbol. Cross tabulation									
Count									
		In the given cube nets, mark opposite faces with the same symbol.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
gender of students	Female	125	10	14	14	3	3	48	217
	Male	115	10	4	11	4	2	53	199
Total		240	20	18	25	7	5	101	416

Table 5. Frequency of responses by grade

Grade * In the given cube nets, mark opposite faces with the same symbol. Cross tabulation									
Count									
		In the given cube nets, mark opposite faces with the same symbol.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
Grade	6th	63	4	3	6	3	0	23	102
	7th	66	8	6	3	0	0	19	102
	8th	59	2	4	11	2	1	24	103
	9th	52	6	5	5	2	4	35	109
Total		240	20	18	25	7	5	101	416

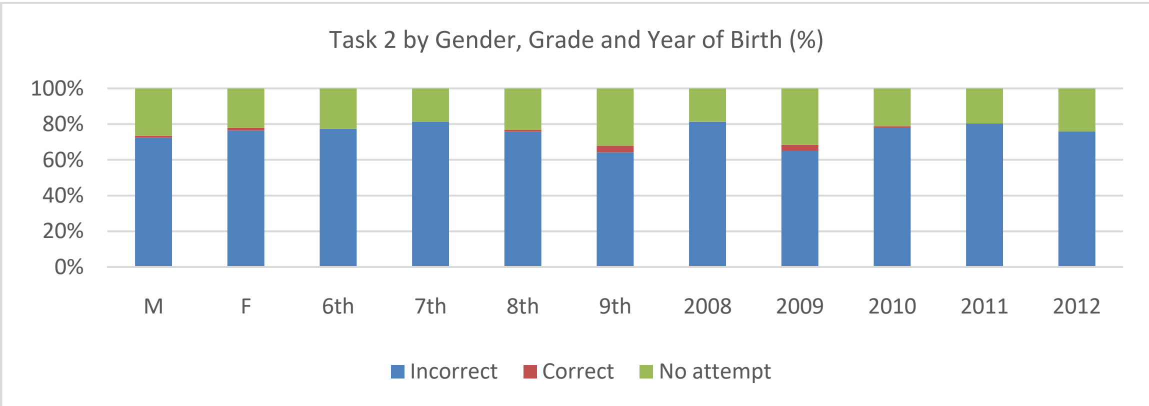
Table 6. Frequency of responses by year of birth

Year of Birth * In the given cube nets, mark opposite faces with the same symbol .Cross tabulation									
Count									
		In the given cube nets, mark opposite faces with the same symbol.							Total
		No correct answers.	One correct answer.	Two correct answers.	Three correct answers.	Four correct answers.	Five correct answers.	Not attempted.	
year of birth	2008	11	1	0	1	0	0	3	16
	2009	58	7	5	5	3	4	38	120
	2010	62	1	7	10	1	1	22	104
	2011	64	8	3	3	0	0	19	97
	2012	45	3	3	6	3	0	19	79
Total		240	20	18	25	7	5	101	416



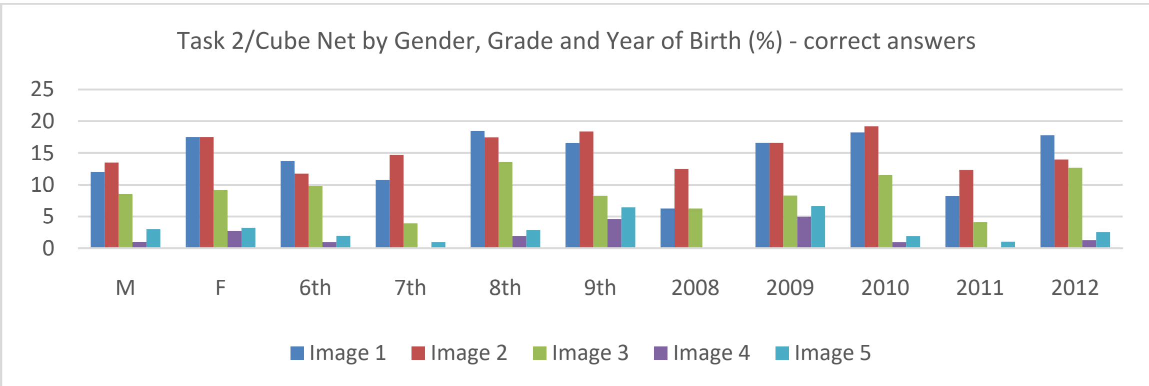
Graph 4. Distribution of all responses for task 2 by all categories expressed in percentages

If we observe the graphically presented data, we do not see any significant deviations between the bars. However, if we observe more precisely, we can conclude that there are very few with all 5 correct answers. Additionally, the percentage of all incorrect answers is concerning.



Graph 5. Distribution of responses classified as correct, incorrect, and no attempt for task 2 by all categories expressed in percentages

If we observe the visually presented data, we must be concerned about the extremely high percentage of incorrect answers and the extremely low percentage of correct answers across all observed categories. Let us note that here we observed the accuracy of the entire task. Since the task consisted of 5 nets, we can also observe the aspect of task completion in relation to each net separately.



Graph 6. Distribution of Correct Answers for Task 2 by All Categories Expressed in Percentages

On this graph, the percentage of correct answers by all categories and for all 5 cube nets is shown. Image 1 and Image 2 have the highest percentages, however, as all percentages are below 20%, we cannot say that these are acceptable results. If we refer again to what is found in our textbooks, a very small number of examples are of this type of task, which means that we can also understand this as a consequence of the way of working, that is, what is available to the students. More precisely, students are at the level of reproduction and are not able to abstract and transfer their knowledge to new situations. They again preferred the known nets (those they recognized in Task 1). That's the highest number of correct answers. The fourth image did not represent a cube net. Very few students noticed this, which is shown by a percentage lower than 5% for all categories. We can assume that students very little and certainly not enough visualize mathematical content. That would be the second assumption derived from this research.

Analysis of Task 3

We presented the obtained data using tables, charts, and figures.

Table 7. Frequency of responses by gender

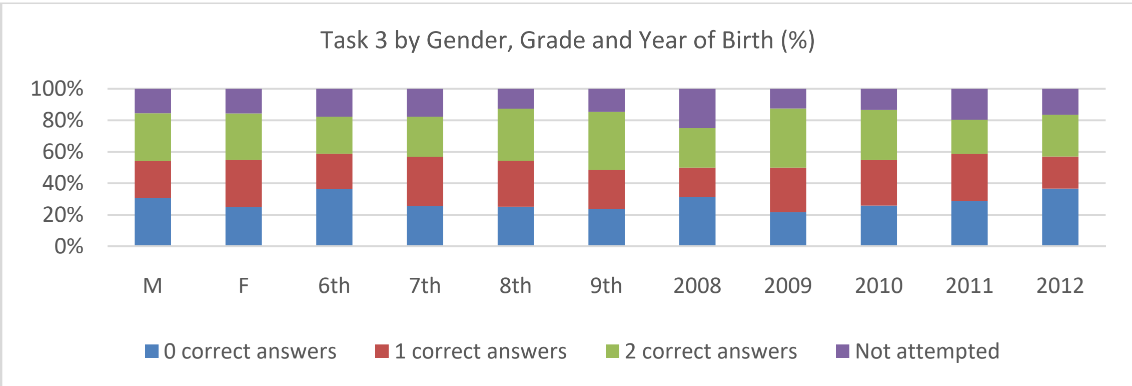
Gender * A figure is given. Create a cube net by drawing one additional square. Cross tabulation						
Count						
		A figure is given. Create a cube net by drawing one additional square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
Gender of students	Female	54	65	64	34	217
	Male	61	47	60	31	199
Total		115	112	124	65	416

Table 8. Frequency of responses by grade

Grade * A figure is given. Create a cube net by drawing one additional square. Cross tabulation						
Count						
		A figure is given. Create a cube net by drawing one additional square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
Grade	6th	37	23	24	18	102
	7th	26	32	26	18	102
	8th	26	30	34	13	103
	9th	26	27	40	16	109
Total		115	112	124	65	416

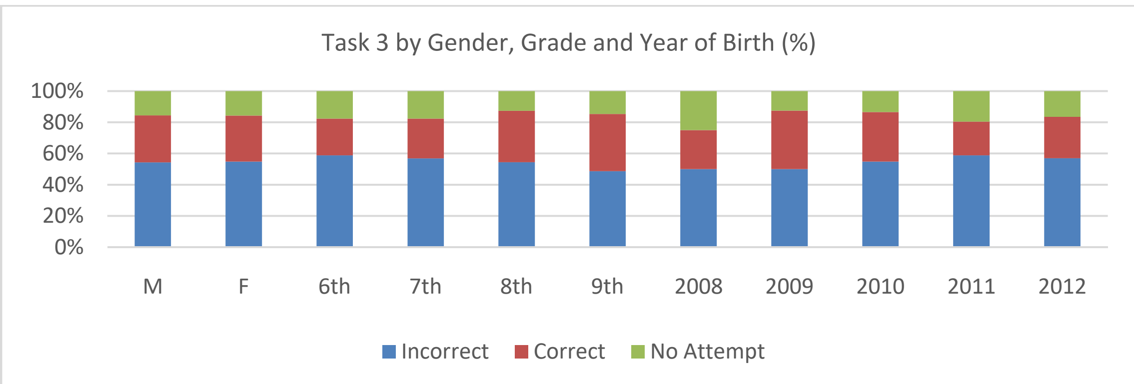
Table 9. Frequency of responses by year of birth

Year of Birth * A figure is given. Create a cube net by drawing one additional square. Crosstabulation						
Count						
		A figure is given. Create a cube net by drawing one additional square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
year of birth	2008	5	3	4	4	16
	2009	26	34	45	15	120
	2010	27	30	33	14	104
	2011	28	29	21	19	97
	2012	29	16	21	13	79
Total		115	112	124	65	416



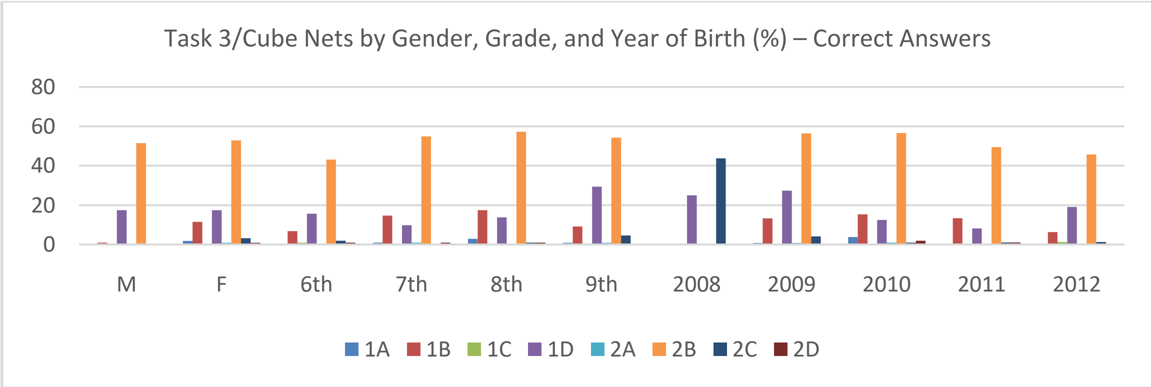
Graph 7. Distribution of all responses for task 3 across all categories expressed in percentages

When observing the graphically presented data, we do not see significant differences between the bars. Considering the requirements of the task, we definitely cannot be satisfied with the achieved results, especially since the second image could have been simplified to one they were familiar with.



Graph 8. Distribution of responses categorized as correct, incorrect, and no attempt for task 3 across all categories expressed in percentages

When analyzing the visually presented data, the high percentage of incorrect answers and those who did not attempt to solve the task across all observed categories is concerning. It is important to note that we examined the accuracy of the entire task here. Since the task consisted of two grids, each with four possible correct answers, we can also analyze the task completion rate in relation to each grid separately. Correct answers are labeled as 1A, 1B, 1C, and 1D for the first image, and 2A, 2B, 2C, and 2D for the second image.



Graph 9. Distribution of correct answers for task 3 across all categories expressed in percentages

This graph displays the percentage of correct answers across all categories. What immediately stands out on the graph is the green color, which in this case represents the answer labeled as 2B.

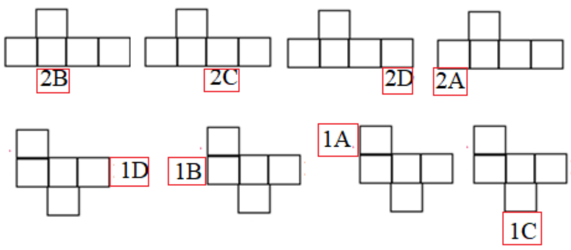


Figure 3. Solutions to task 3 sorted by frequency of answer selection

The selection of 2B represents the most common choice, with 2C being similar, thus further confirming the assumption about the influence of the most prevalent patterns during free-choice problem-solving methods. The selections 1D and 1B are similar to the most common case because they feature four squares in a row, which also, in a way, supports the previously mentioned hypothesis.

Therefore, this task also suggests the assumption that the most prevalent scenario influences the choice of problem-solving methods in free-choice tasks.

Analysis of Task 4

The obtained data are presented using tables, graphs, and figures.

Table 10. Frequency of responses by gender

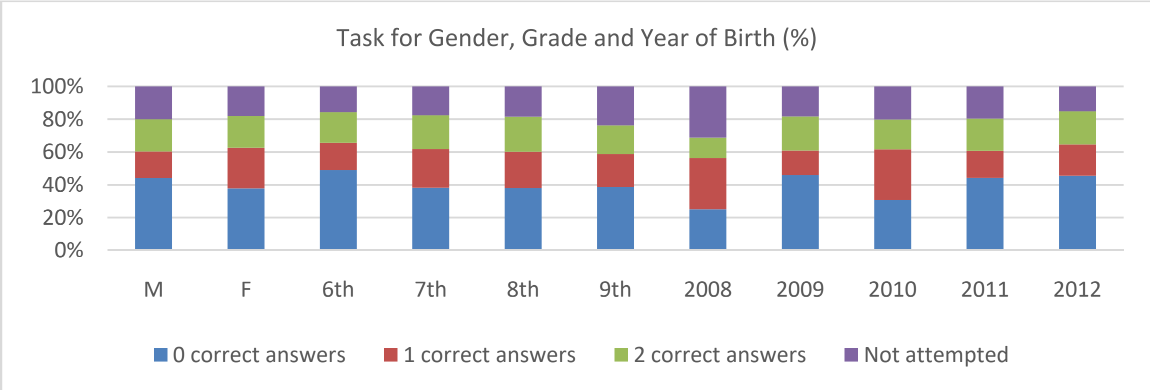
Gender of students * A figure is given. Create a cube net by crossing one square. Cross tabulation						
Count						
		A figure is given. Create a cube net by crossing one square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
gender	Female	82	54	42	39	217
	Male	88	32	39	40	199
Total		170	86	81	79	416

Table 11. Frequency of responses by grade

Grade * A figure is given. Create a cube net by crossing one square. Cross tabulation						
Count						
		A figure is given. Create a cube net by crossing one square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
Grade	6^{th}	50	17	19	16	102
	7^{th}	39	24	21	18	102
	8^{th}	39	23	22	19	103
	9^{th}	42	22	19	26	109
Total		170	86	81	79	416

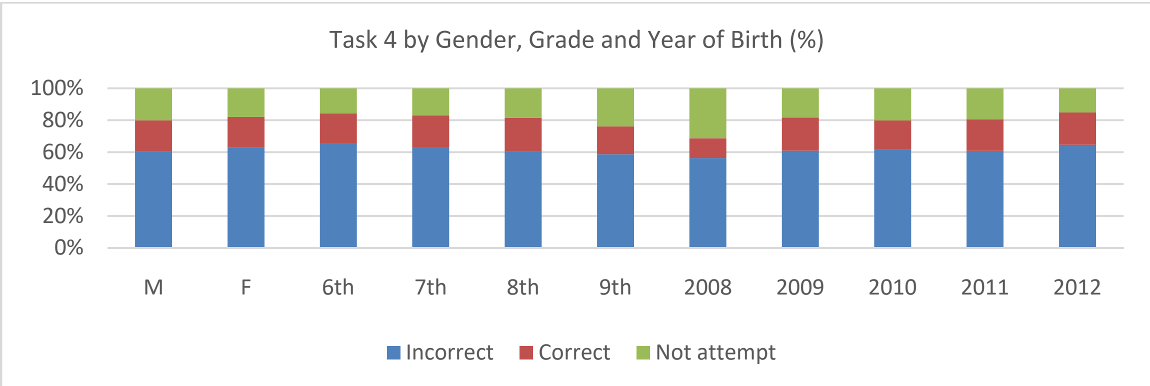
Table 12. Frequency of responses by year of birth

Year of Birth * A figure is given. Create a cube net by crossing one square. Cross tabulation						
Count						
		A figure is given. Create a cube net by crossing one square.				Total
		No correct answers.	One correct answer.	Two correct answers.	Not attempted.	
year of birth	2008	4	5	2	5	16
	2009	55	18	25	22	120
	2010	32	32	19	21	104
	2011	43	16	19	19	97
	2012	36	15	16	12	79
Total		170	86	81	79	416



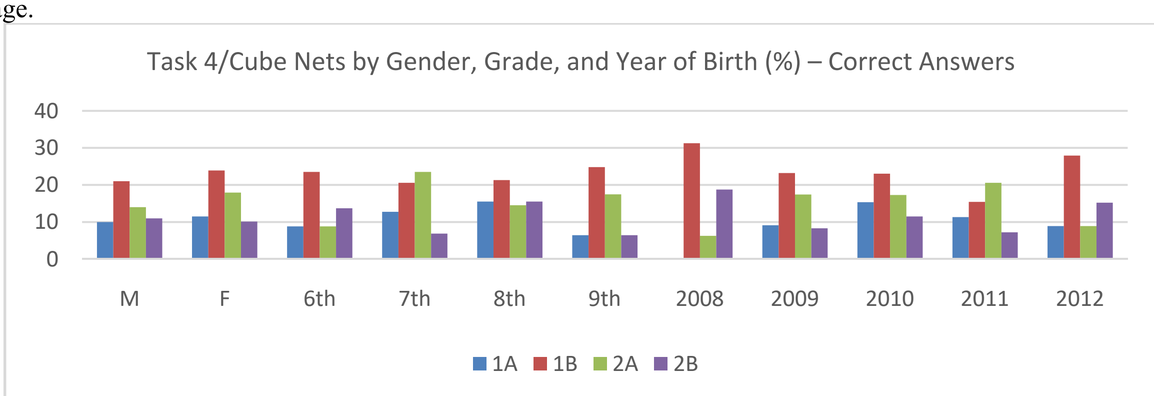
Graph 10. Distribution of all responses for task 4 across all categories expressed in percentages

When observing the graphically presented data, we do not see significant deviations between the bars. Considering the requirements of the task, we definitely cannot be satisfied with the achieved results, especially since the first image could have been simplified to one they were familiar with.



Graph 11. Distribution of responses categorized as correct, incorrect, and no attempt for task 4 across all categories expressed in percentages

When observing the visually presented data, we are once again concerned by the high percentage of incorrect answers and those who did not attempt to solve the task across all observed categories. It is important to note that we examined the accuracy of the entire task here. Since the task consisted of two nets, each with two possible correct answers, we can also analyze the task completion rate in relation to each net separately. Correct answers are labeled as 1A, 1B for the first image, and 2A, 2B for the second image.



Graph 12. Distribution of correct answers for task 4 across all categories expressed in percentages

This graph displays the percentage of correct answers across all categories. What immediately stands out on the graph is the orange color, which in this case represents the answer labeled as 1B.

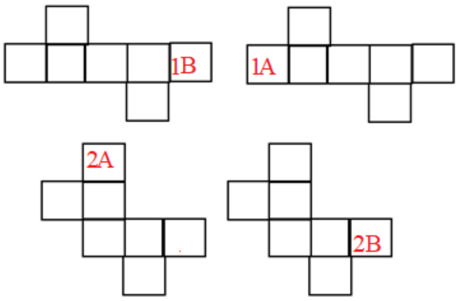


Figure 4. Solutions to task 4 sorted by frequency of answer selection

The selection of 1B is the most common, but the percentage is still quite low for this type of task. This leads us to hypothesize that students rarely work on tasks of this type, which is why they solve them at such a low percentage. This would be the assumption arising from this task.

Analysis of Task 5

The obtained data are presented using tables, graphs, and images.

Table 13. Frequency of responses by gender

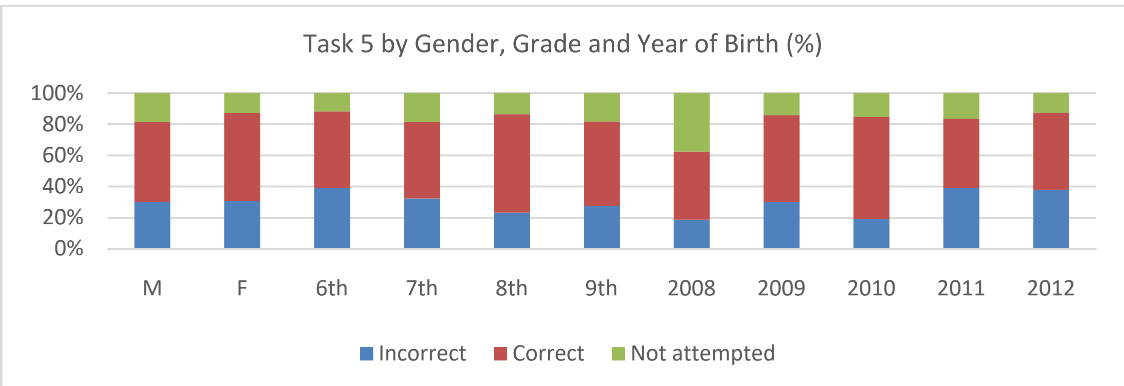
Gender of students * A figure is given. Create a cube net by moving one square from one position to another. Cross tabulation					
Count					
		A figure is given. Create a cube net by moving one square from one position to another.			Total
		Incorrect answer	Correct answer	Not attempted	
gender of students	Female	67	122	28	217
	Male	60	102	37	199
Total		127	224	65	416

Table 14. Frequency of responses by grade

Grade * A figure is given. Create a cube net by moving one square from one position to another. Cross tabulation					
Count					
		A figure is given. Create a cube net by moving one square from one position to another.			Total
		Incorrect answer	Correct answer	Not attempted	
Grade	6th	40	50	12	102
	7th	33	50	19	102
	8th	24	65	14	103
	9th	30	59	20	109
Total		127	224	65	416

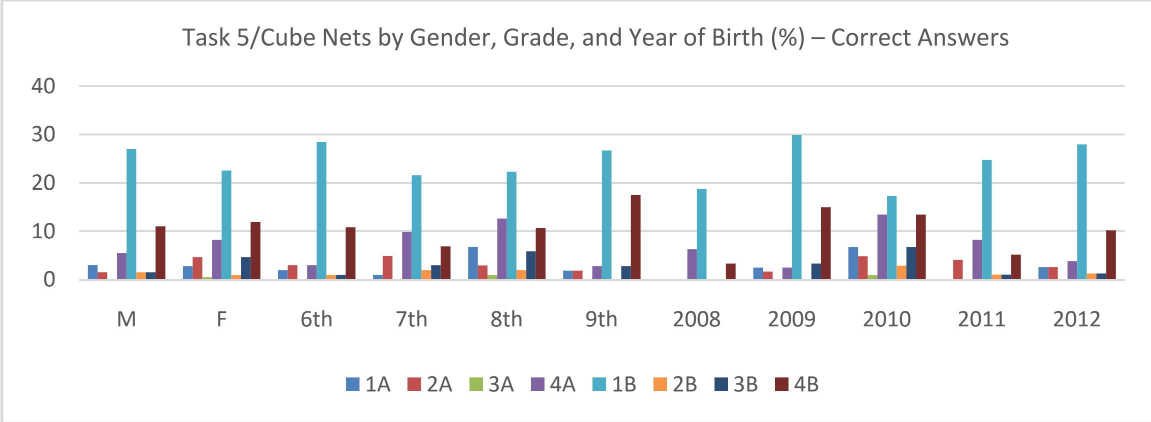
Table 15. Frequency of responses by year of birth

Year of Birth * A figure is given. Create a cube net by moving one square from one position to another. Cross tabulation					
Count					
		A figure is given. Create a cube net by moving one square from one position to another.			Total
		Incorrect answer	Correct answer	Not attempted	
year of birth	2008	3	7	6	16
	2009	36	67	17	120
	2010	20	68	16	104
	2011	38	43	16	97
	2012	30	39	10	79
Total		127	224	65	416



Graph 13. Distribution of all responses for task 5 across all categories expressed in percentages

When observing the graphically presented data, we do not see significant deviations between the bars. Considering the requirements of the task, we definitely cannot be satisfied with the achieved results. However, compared to the other tasks, this task has the highest percentage of correct answers. Unlike the previous four tasks, this one only had one net. There was no option for a partially correct task as in the previous ones. However, this task still had 8 possible solutions, which we labeled as: 1A, 1B, 1C, 1D, 2A, 2B, 2C, 2D.



Graph 15. Distribution of correct answers for task 5 across all categories expressed in percentages

This graph displays the percentage of correct answers across all categories. What immediately stands out on the graph is the orange color, which in this case represents the answer labeled as 1B.

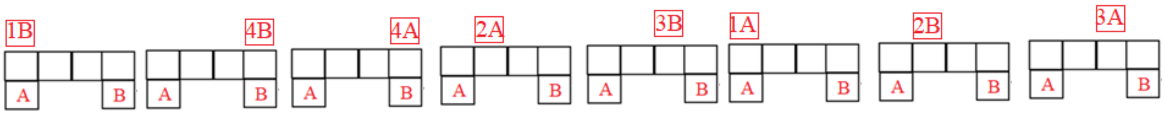


Figure 5. Solutions to task 5 sorted by frequency of answer selection

The selection of 1B is the most common, followed by 4B. One possible assumption is that most of these students are right-handed and therefore start from the right side, move to the beginning, and then to the end. This would be the assumption arising from this task.

Conclusion

One additional observation, which is not typical in the application of mandatory written knowledge assessments, is the comments that students wrote on their papers. The common denominator of all these comments is that the students emphasized that they did not have to complete the tasks (since, clearly, they were not conditioned by grades, as they are used to). There were also comments mentioning that this is an IQ test and that they wanted feedback on "how smart they are," or, in contrast, comments stating that there is no need to measure their IQ because they already know "they are dumb." Fortunately, there were not many such comments, but they are certainly an indication that something needs to change in our education system. Next, students mentioned that they did not have to solve these tasks and left them blank or simply doodled on them. Some even wrote "I have no idea" next to each task. Regarding the research tasks set, we came to the following conclusions: The tasks we selected to be part of the research instrument provided us with the answers we sought. Comparing what is offered in school textbooks and workbooks influenced the way students think when solving tasks. They mostly narrowed their choices to what they had already seen, based on how they had done it during lessons. Only a few deviated from this. As for the variables by which we analyzed student work, we did not notice any significant deviations. Another thing we noticed is the need to work on student motivation when it comes to some non-compulsory tasks. Thus, it is necessary to build their awareness about the importance of their effort and work beyond the segment that results in a grade. Additionally, we sensed the students' need to express their discontent when they were anonymous and knew that no behavior would be sanctioned. This leads us to the conclusion that students need to express their opinions publicly without fear of consequences if their opinion does not align with the surrounding environment. It is also necessary to work on making students aware that although they have the right to their opinion, this does not mean they can misuse it. We believe it would be interesting to conduct a similar study with older students—high school students—and we have already started conducting such research.

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Statement of Competing Interests

The authors have no competing interests.

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