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Research Article

AN ATTEMPT OF MATHEMATICAL MODELLING OF RAINFALL. CASE OF NDJAMENA, CHAD. PERIOD FROM 1951 TO 2010 INCLUDED

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Abstract

The frequent occurrence of droughts and floods in some sahelian countries in general and particularly in Ndjamena have serious impacts in the daily life of inhabitants. This situation is generated by time-space uncontrollable variabilities of the durations of the dry and rainy seasons. Sometimes, the rainy seasons are shortened, and inversely. These variabilities are usually very catastrophic for the environment. Whence the necessity of the present investigation of the monthly averages of rainfall registered at the international airport of Ndjamena over sixty years, i.e. two successive climatic periods, 1951-1980 and 1981-2010. Each climatic period has been divided into six equal sub periods of five years. For each one, numbers of rainy days, cumulative rainfalls with their standard deviations, average sub period monthly rainfalls and the probabilities of their occurrences have been determined and analyzed. This study confirms the high time variabilities of the rainfall. Moreover, comparison of the two climatic periods has revealed the time degradation of the rainfall regime in the city. This degradation has been quantified. Using the least square approach, the time trend of the decreasing rainfall regime has been modeled with a negative line regression.

Keywords: Mathematical modelling, Rainfall, Sub periods, Monthly averages of rainfall, Cumulative rainfalls, Standard deviation, Cloud of experimental points, Least square method.

INTRODUCTION

1. Position of the problem

Chad is a tropical Sahelian inside-land country situated almost at the heart of Africa. It's very poor coverage by vegetation and extremely rare hydrography make its climate very rude. One of the former biggest African lakes which provides rains to these sub regional countries is today about to disappear. Within 50 years period, its territory has been reduced from 25000 square kilometers to even less than 2000 square kilometers. The main rivers crossing part of the country, Chad, through its capital, Ndjamena, seem to get dry from year to year. With the increase of the populations, one passively assists to an intensive the destruction of the remaining trees which could bring the environment positive impacts. These particular problems and the climate change in general make the rainfall regime uncontrollable, i.e. unmanageable. Moreover, the frequent lack of regular chronological series of data complicates the situation and makes investigations very difficult. Recall that Chad is a tropical country with two seasons: dry and rainy ones. The dry season is always longer than the rainy one. Formerly the dry season could cover up to nine months in a year in many localities of the country. Nowadays, some changes have affected the durations and intensities of these seasons. Investigations on these changes have been carried out by some researchers. For example, the rank of sahelian latitudinal rainfall gradient is 1mm/km in the horizontal plane, (Lebel *et al.,* 2003). The drastically reduction of the rainfall during the period from 1970 to 1990 has led many African Sahelian countries to unprecedent humanitarian catastrophes and one should always recall Ethiopia, (Le Barbé *et al.,* 1997). The impacts of human activities should not be negligible as they could have negative consequences on the rainfall regime as proved by some researchers, (Charney *et al.,* 1975). As the situation is always going deeper, few countries including Chadin the sub region have started flying with special equipped planes in the clouds to modify their structure introducing into them appropriate substances in order to stimulate rains when needed. For Chad, some data from these expeditions has been treated and results published, (Njipouakouyou, October 2017; Njipouakouyou, November 2017). For more extension purpose, a sample and easily applicable method of rainfall forecast using allover daily observed atmospheric parameter was developed for the case of Ndjamena, (Njipouakouyou *et al.,* 2019). More recent and detailed investigations on the same item was carried by Moussa in his M.Sc. thesis, (Moussa, 2023). It is obvious that for better managing the rainfall regime in Chad in general and Ndjamena in particular, we must first investigate the time space trends of rain. Whence the importance of this paper. This paper is divided into five parts. The first and present one is the introduction. Here the problematic of our work is exposed with some recent literature on the same topic from different authors. The second part presents and discusses the data and methodology used for the treatments. The third one exposes and analyzes the obtained results. The fourth part is the conclusion and recommendations while the fifth part indicates the references used by the authors for this investigation.

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DATA AND METHODOLOGY

Data

The used data for this work comes from the observations of the rainfall at the meteorological station of the international airport of Ndjamena, Chad. It covers the period from 1951 to 2010 included. This data is the monthly average of rainfall for that period. As an international airport, no doubt that this station is equipped with modern instruments and the workers are well-trained. Therefore, the probability of good representativity of these data has no doubt. Thus, the obtained results should be highly appreciated. Concerning the geographical position of Chad, note that it is situated at the center of the African continent. It is limited with the following countries: Libya in the north, Soudan in the east, Central African Republic in the south-east, Cameroon in the south-west, Nigeria and Niger in the west. Ndjamena is the main city of the country, the political capital where are concentrated all the government institutions, international and foreign representations. It is situated at the south-west almost on the bank of the Chari river, the border with Cameroon, conferred Figure 1.

Figure 1. Map of Chad and position of its capital, Ndjamena

Methodology

The period 1951-2010 corresponds to two climatic periods. According to the World Meteorological Organization, WMO, for a period to be qualified as climatic, it must contain at least thirty years. Hence results issued from the investigation should also be considered as climatic. The two climatic period are 1951-1980 and 1981-2010. For each period, monthly averages of rainfall are divided into six sub groups of equal duration of five years. For a sub period the number n_R of rainy days are determined, the cumulative rainfalls, R, and corresponding standard deviations, S, are calculated, the sub period averages, $\overline{R}$ and the probabilities, p, of their occurrences were also determined. All these characteristics are analyzed and compared between sub periods. This analysis has enabled us to put out the periods when the fluctuations of the rainfalls were very considerable, and inversely.

Going further, the time trends of the rainfalls in the whole period of investigation also on the base of the least square method are made. For this analysis cumulative monthly rainfalls, R, over the entire period are calculated and experimental points R(t) plotted in a coordinates plane to find the forma of the relationship R(t) between the monthly rainfall (in mm) and the time t (in years). The obtained model should enable us to highlight either rainfalls in Ndjamena increase or decrease from year to year. No doubt that all these results will help the authorities to take adequate, coherent and on time decisions to avoid eventual catastrophes to occur in the city.

RESULTS OF DATA TREATMENTS AND THEIR ANALYSIS

These results are presented in tabular forms. In these tables, n indicates the number of rainy days in the considered period, R (in mm) and S (in mm) - the rainfalls and standard deviations, $\overline{R}$(in mm) – monthly averages of rainfall and p-their probabilities of occurrence. P_{ij} indicates the climatic sub period where i stands for the climatic period and j for the sub period. So P_{24} corresponds to the second climatic period and the fourth sub period.

The sub period statistical characteristics of the rainfall during the first climatic period, 1951-1980, are shown in Table 3.1. It indicates that January, February, November and December were completely dry season. And because of the severe dryness and sandy structure of the soil, any medium wind should able to fill the atmosphere with all kinds of particles, sand between others. Inhabitants should be very careful because they are very openly exposed to many diseases such as eyes diseases, cardiovascular diseases, dermatological diseases, just to name a few.

In general, the rainy season very weekly starts in April and ends in October. During these months the number of rainy days oscillates around 3 for all sub periods. From by the end of May till September, the number of rainy days climbs to 5 during all the sub periods of the first climatic period. At the beginning of the rainy season, the rainfall increased to its maximum value in July-August when it begins decreasing. It is obvious that during these months, people should be very attentive to the high risk of floods because of the rainfall and the quality of the soil not appropriate for rapid infiltration of water into the ground. By this time, risks of other diseases occur, particularly malaria and cholera. Examining the general time trend of the sub-period monthly rainfalls during the first climatic period, one easily comes to the conclusion that from year to year they were decreasing, particularly in August. This trend is perfectly seen when considered the last colon of Table 3.1 where the sub periods cumulative annual rainfalls are presented. The probability of occurrence of the sub period monthly rainfalls during the rainy season was allover 1, meaning that there was no doubt in their occurrences. During the rainy season, the rainiest month was August with an absolute maximum value of rainfall of 1368.1 mm and absolute minimum value of 852.8 mm registered during the first, P_{11}, and sixth, P_{16}, sub periods corresponding respectively to the laps of time 1951-1955 and 1975-1980. The less rainy periods coincide with the beginning, in April, and the end, in October, of the rainy season. During these periods the rainfalls were included in the interval 10.5 mm (P_{11}) -107.8 mm (P_{14}) for the beginning and 18.6 mm (P_{12}) – 203.7mm (P_{11}) for the end, respectively.

Table 1. Statistical characteristics of the sub period monthly rainfalls in Ndjamena during the first climatic period, 1951-1980

Periods	Para	Janv.	Feb.	Marc	April	May	June	July.	Aug	Sept.	Oct.	Nov.	Dec.	Cumul
	n	0	0	0	2	5	5	5	5	5	5	0	0	
	R	0	0	0	10.5	156.4	377.8	741.2	1368.1	607.3	203.7	0	0	3279.8
P_{11}	S	0	0	0	3.8	22.6	30.4	84.5	136.7	56.9	28.1	0	0	109.9
	$\bar{R}$	0	0	0	2.1	31.3	75.6	148.2	273.6	212.5	40.7	0	0	656.0
	p	0	0	0	0.4	1.00	1.00	1.00	1.00	1.00	1.00	0	0	
	n	0	0	0	4	5	5	5	5	5	4	0	0	
	R	0	0	0	19.3	217.0	334.6	942.1	1200.2	676.0	18.6	0	0	3610.1
P_{12}	S	0	0	0	4.3	43.8	38.8	59.8	98.2	27.8	4.9	0	0	172.9
	$\bar{R}$	0	0	0	3.9	43.4	66.9	188.4	240.0	135.2	3.7	0	0	722.0
	p	0	0	0	0.8	1.00	1.00	1.00	1.00	1.00	0.8	0	0	
	n	0	0	3	4	5	5	5	5	5	4	0	0	
	R	0	0	6.5	55.8	31.1	313.1	693.7	1109.0	564.3	58.7	0	0	2778.1
P_{13}	S	0	0	1.4	10.5	5.2	22.3	41.7	67.4	49.1	7.3	0	0	127.2
	$\bar{R}$	0	0	1.3	11.2	6.2	62.6	138.7	221.8	112.9	11.7	0	0	555.6
	p	0	0	0.6	0.8	1.00	1.00	1.00	1.00	1.00	0.8	0	0	
	n	0	0	0	4	5	5	5	5	5	4	0	0	
	R	0	0	0	107.8	167.2	336.2	748.5	1031.5	388.1	106.9	0	0	2822.4
P_{14}	S	0	0	0	17.7	19.9	38.8	35.0	76.9	29.4	22.5	0	0	59.2
	$\bar{R}$	0	0	0	21.6	33.4	67.2	149.7	206.3	77.6	21.4	0	0	564.5
	p	0	0	0	0.8	1.00	1.00	1.00	1.00	1.00		0	0	
	n	0	0	0	4	5	5	5	5	5	4	0	0	
	R	0	0	0	12.3	90.4	184.8	779.2	58.3	483.1	102.4	0	0	2417.4
P_{15}	S	0	0	0	2.4	9.3	36.5	72.0	68.1	34.8	27.9	0	0	140.2
	$\bar{R}$	0	0	0	2.5	18.1	37.0	155.8	171.7	96.6	20.5	0	0	483.5
	P	0	0	0	0.8	1.00	1.00	1.00	1.00	1.00	0.8	0	0	
	N	0	0	0	3	5	5	5	5	5	4	0	0	
	R	0	0	0	70.8	259.6	347.4	678.4	852.8	387.6	203.5	0	0	3149.8
P_{16}	S	0	0	0	13.5	24.7	23.3	62.5	71.5	23.3	33.6	0	0	87.1
	$\bar{R}$	0	0	0	14.2	51.7	69.5	135.7	170.6	7.5	40.7	0	0	630.0
	p	0	0	0	0.6	1.00	1.00	1.00	1.00	1.00	0.8	0	0	
	n_P	0	0	1	6	6	6	6	6	6	6			
	R_P	0	0	6.5	276.5	921.7	1893.9	4583.1	6419.9	3106.4	693.8	0	0	18056.6
P_1	S_P	0	0	2.7	39.1	83.0	67.5	94.8	200.4	118.5	75.3	0	0	422.5
	$\bar{R}_P$	0	0	1.1	46.1	153.6	315.7	763.9	1070.0	517.7	115.6	0	0	3009.6
	P_P	0	0	0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0	0	

Table 2. Statistical characteristics of the sub period monthly rainfalls in Ndjamena during the second climatic period, 1981-2010

Periods	Para	Janv.	Feb.	Marc	April	May	June	July.	Aug	Sept.	Oct.	Nov.	Dec.	Cumul
	n	0	0	0	3	5	5	5	5	5	2	0	0	
	R	0	0	0	56.3	77.5	142.7	568.1	650 ;1	278.8	29.1	0	0	1834.4
P_{21}	S	0	0	0	20.6	15.9	8.6	58.6	56.7	32.5	12.0	0	0	83.9
	$\bar{R}$	0	0	0	11.3	15.5	28.5	113.6	130.0	55.8	5.8	0	0	366.9
	p	0	0	0	0.6	1.00	1.00	1.00	1.00	1.00	0.4	0	0	
	n	0	0	0	1	5	5	5	5	5	5	0	0	
	R	0	0	0	6.1	149.0	207.0	847.5	730.1	426.0	108.4	0	0	2550.5
P_{22}	S	0	0	0	2.7	31.0	32.2	62.3	48.7	64.0	34.1	0	0	117.9
	$\bar{R}$	0	0	0	1.2	29.8	41.4	169.5	146.0	85.2	21.7	0	0	510.1
	p	0	0	0	0.2	1.00	1.00	1.00	1.00	1.00	1.00	0	0	
	n	0	0	1	5	4	5	5	5	5	5	0	0	
	R	0	0	1.2	51.0	141.4	207.4	41.8	996.9	482.9	76.6	0	0	2550.4
P_{23}	S	0	0	0.5	7.6	32.0	11.1	16.8	61.6	42.7	11.8	0	0	38.9
	$\bar{R}$	0	0	0.2	10.2	28.3	41.5	148.4	199.4	96.6	15.3	0	0	510.1
	p	0	0	0.2	1.00	0.8	1.00	1.00	1.00	1.00	1.00	0	0	
	n	0	0	0	3	4	5	5	5	5	5	1	0	
	R	0	0	0	39.3	99.7	278.1	1035.0	1047.1	343.3	165.2	25.6	0	2801.7
P_{24}	S	0	0	0	7.5	23.3	35.8	84.7	51.9	34.0	13.2	11.4	0	152.3
	$\bar{R}$	0	0	0	7.9	19.9	55.6	207.0	209.4	8.7	33.0	5.1	0	560.3
	p	0	0	0	0.6	0.8	1.00	1.00	1.00	1.00	1.00	0.2	0	
	n	0	0	0	2	4	5	5	5	5	4	0	0	
	R	0	0	0	40.3	197.5	245.7	717.8	935 ;1	615.1	134.9	0	0	3043.1
P_{25}	S	0	0	0	17.1	43.8	4.8	28.2	61.4	80.4	31.4	0	0	63.3
	$\bar{R}$	0	0	0	8.1	39.5	49.1	143.6	187.0	123.0	27.0	0	0	608.6
	p	0	0	0	0.4	0.8	1.00	1.00	1.00	1.00	0.8	0	0	
	n	0	0	0	0	5	5	5	5	5	5	1	0	
	R	0	0	0	0	88.7	309.0	969.4	1128.0	451.3	149.8	17.5	0	3076.3
P_{26}	S	0	0	0	0	19.6	36.5	32.8	75.5	44.0	17.4	7.4	0	77.1
	$\bar{R}$	0	0	0	0	17.7	61.8	193.9	225.6	92.3	30.0	3.3	0	615.3
	p	0	0	0	0	1.00	1.00	1.00	1.00	1.00	1.00	0.2	0	
	n_P	0	0	1	6	6	6	6	6	6	6	2	0	
	R_P	0	0	1.2	193.0	753.8	1389.9	4879.6	5487.3	2607.4	664.0	42.1	0	15856.4
P_2	S_P	0	0	0.5	23.5	45.4	59.1	172.7	186.7	116.9	50.8	11.2	0	456.9
	$\bar{R}_P$	0	0	0.2	32.2	125.6	231.7	813.3	914.6	434.6	110.7	7.0	0	2642.7
	P_P	0	0	0.2	0.8	1.00	1.00	1.00	1.00	1.00	1.00	0.3	0	

Tableau 3. Statistical characteristics of rainfall for both climatic periods

Périodes	Para	Janv.	Fev.	Mars	Avril	Mai	Juin	Juil.	Août	Sept.	Oct.	Nov.	Dec.	Cumul
	n_P	0	0	1	6	6	6	6	6	6	6			
	R_P	0	0	6.5	276.5	921.7	1893.9	4583.1	6419.9	3106.4	693.8	0	0	18056.6
P_1	S_P	0	0	2.7	39.1	83.0	67.5	94.8	200.4	118.5	75.3	0	0	422.5
	$\overline{R}_P$	0	0	1.1	46.1	153.6	315.7	763.9	1070.0	517.7	115.6	0	0	3009.6
	P_P	0	0	0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0	0	
	n_P	0	0	1	6	6	6	6	6	6	6	2	0	
	R_P	0	0	1.2	193.0	753.8	1389.9	4879.6	5487.3	2607.4	664.0	42.1	0	15856.4
P_2	S_P	0	0	0.5	23.5	45.4	59.1	172.7	186.7	116.9	50.8	11.2	0	456.9
	$\overline{R}_P$	0	0	0.2	32.2	125.6	231.7	813.3	914.6	434.6	110.7	7.0	0	2642.7
	P_P	0	0	0.2	0.8	1.00	1.00	1.00	1.00	1.00	1.00	0.3	0	
P_3- $P_{1\backslash}$		0	0	-5.3	-83.5	-167.9	-504.0	296.5	-932.6	-499.0	-29.8	42.1	0	-2198.2

The values of the standard deviation show that the sub period variability of the rainfalls was all over one order less the corresponding values of the main parameter. High values of the standard deviation during the rainiest period indicate the relatively high variability of the rainfall. The statistical characteristics for the whole climatic period are found by the end of Table 3.1. It indicates an amount of 18056.6 mm cumulative rainfall with a standard deviation of 422.5 mm. The rainiest months were July, August and September with respectively 4583.1, 6419.9 and 3106.4 mm. The sub period statistical characteristics of the rainfall for the second climatic period are in table 2.

Qualitatively going through Table 2, it comes that during the rainiest time the sub period numbers of monthly rainy days were kept to 5. Other time tendencies of rainfalls relatively to the dry and rainy months were remained without change.

Quantitative examination of Table 2 reveals some significant differences. Firstly, the values of corresponding sub period monthly statistical characteristics of the rainfall are greater during the first period than during the second.

At the beginning and end of the rainy season, sub period monthly values of the rainfalls varied in the next intervals 6.1 mm (P_{22}) – 56.3 mm (P_{21}) and 29.1 mm (P_{21}) - 165.2 mm (P_{24}), respectively. The maximum values of rainfalls were usually registered in August. They were contained in the interval 650.1 mm (P_{21}) – 1128.0 mm (P_{26}). Corresponding values of the standard deviation were significantly high in the second climatic period than in the first. This tendency indicates that the rain variability was higher in the secand period than in the first. Moreover, the corresponding values of the sub period monthly rainfall were less in the second climatic period than in the first, meaning that the second climatic period was drier than the first.

Statistical characteristics of the rainfall for both climatic periods in a whole are in Table 3.3. It contains the monthly values of the following parameters, the numbers of rainy days, n_P, the rainfalls, R_P, their standard deviations, S_P, the monthly average rainfall, $\overline{R_P}$, and the probabilities of their realization, p_P. Table 3.3 also contains the differences P_2-P_1 between the monthly rainfalls of the second, P_2, and first, P_1, climatic periods. Comparative analysis of both climatic periods indicates that the first one was rainier with lower time variabilities than the second one. This conclusion is coherent with the negative values of those differences almost all over. These results confirm the today global climate change moving toward its deterioration. Particularly for this investigation, this deterioration concerns the rainfall regime in Ndjamena during the considered laps of time.

This deficit was important from May with -167.9 mm to September with -499.0 mm, the maximum value of -932.6 mm been registered in August. An attempt of time modeling of the deterioration of the monthly rainfalls in Ndjamena for the whole period using the least square method in order to obtain the functional relationship R(t) between the rainfalls and the time in months was made. To facilitate the computation processes, an auxiliary variable, t, was introduced, with t=0 for the month of June. Thus, the sums R_i, i = 1, 2, …, 12, of monthly rainfalls were found. These values are in Table 3.4.

In order to find the form of relationship R(t), a cloud of experimental points was plotted in coordinates system toR, t in the abscissa and R in the ordinates, conferred Figure 1.

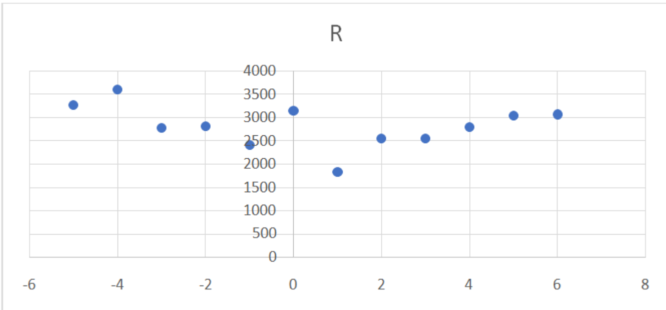

Figure 1.Cloud of experimental points R_i.

In fact, Table 3 also indicates intermediary computations when applying the least square method

The analysis of Figure 1 leads to a linear relationship between variables t and R. Thus, we may write the formula

$$R(t) = at + b, \quad (1)$$

and based on the least square method we have the values of the coefficients a and b and finally the searched function:

$$R(t) = -33.1t + 3252.7. \quad (2)$$

In Table 3.4, the last column indicates the degree of fitness, $\varepsilon_i = R_{th}-R_i$, of the established model. Despite the fact that this degree of fitness is one order lower its corresponding main parameter, their relatively high values confirm the variability of rainfall in general.

Conclusion and recommendations

The present study has clearly put out the degradation of the rainfall regime in Ndjamena during the considered period. With the ongoing today destruction of the environment due to the increasing demography, no doubt that this degradation should be moving deeper. This is also remarkable through the high temp of the drastically permanent dryness of almost all the rivers in the region. To avoid future humanitarian catastrophes, authorities should take urgent measures leading to the substantial reduction of this phenomena. Moreover, they should pay more attention to the education of inhabitants to the protection of the environment. At last, the authorities should manage green spaces all over the city.

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